Roll No :
Date : 2026-02-15

School : ACHIEVERS FOUNDATION
Assessment : Units and Measurements
Subject : physics
Class : XI

Time : 00
Marks: 210

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1 Write the order of following length in metres : **2**

(i) R adius of earth.
(ii) The height of average man.
(iii) Thickness of sheet of paper.
(iv) The radius of hydrogen atom.

Ans :(i) 6.4×10^6 m
(ii) 1.8×10^0 m
(iii) 1×10^{-4} m
(iv) 5×10^{-11} m

2 Write the order of following : **2**

(i) Mass of a housefly.
(ii) Mass of average man.
(iii) Mass of an electron.
(iv) Mass of earth.

Ans :(i) 1×10^{-4} kg
(ii) 7×10^1 kg
(iii) 9.1×10^{-31} kg
(iv) 6×10^{24} kg

3 Write the order of following intervals in seconds : **2**

(i) Time between two heart beats.
(ii) Time of earth's revolution.
(iii) Time of earth's rotation.
(iv) Human life.

Ans :(i) 1×10^0 s
(ii) 3×10^7 s
(iii) 8.6×10^4 s
(iv) 2×10^9 s

4 Fill in the blanks by suitable conversions: **2**

(i) $G = 6.67 \times 10^{-11}$ Nm^2kg^{-2} = ... $cm^2\ s^{-2}\ g^{-1}$

(ii) 1 m = ... light year

(iii) 6 ms^{-2} = ... kmh^{-2}.

(iv) 1 kg m^2s^{-2} = ... g $cm^2\ s^{-2}$.

Ans :(i) 6.67×10^{-8} $cm^2\ g^{-1}\ s^{-2}$

(ii) 1.06×10^{-16}

(iii) 7.776×10^4

(iv) 10^7

5A calorie is a unit of heat or energy and it equals about 4.18 J where 1 J = 1 kg $m^2\ s^{-2}$. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the unit of time is γ s. Show that a calorie has a magnitude $4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^2$ in terms of the new units. **2**

Ans :

1 calorie = 4.18 J = 4.18 kg $m^2\ s^{-2}$

As new unit of mass = α kg, hence in terms of new unit 1 kg = $\frac{1}{\alpha} = \alpha^{-1}$.

Similarly, in terms of the new unit of length 1 m = β^{-1} or 1 $m^2 = \beta^{-2}$

In terms of the new unit of time 1 s = γ^{-1}

∴ 1 $s^{-2} = (\gamma^{-1})^{-2} = \gamma^2$

Hence, 1 calorie = 4.18 kg $m^2\ s^{-2}$ = $4.18(\alpha^{-1})(\beta^{-2})(\gamma^2) = 4.18\alpha^{-1}\beta^{-2}\gamma^2$

6 Check whether equation $F.S = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ is dimensionally correct, where m is mass of the body, v its final velocity, u its initial velocity, F is force applied and S is the distance moved. **2**

Ans : $F.S = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

L.H.S = $[ML^2T^{-2}]$

R.H.S = $\frac{1}{2}[M][LT^{-1}]^2 - \frac{1}{2}[M]\ [LT^{-1}]^2$

$[ML^2T^{-2}] = [ML^2\ T^{-2}] - [ML^2\ T^{-2}]$

L.H.S = R.H.S

Which is dimensionally correct.

7 Find the area of the circle of radius 3.458 cm upto correct significant figures. **2**

Ans : Area = $\pi r^2 = 3.141 \times (3.458)^2$

$= 3.141 \times 11.96$

$= 37.5664 = 37.57$ cm^2

8 Rule out or accept the following formulae for kinetic energy on the basis of dimensional arguments: **2**

(i) $\frac{3}{16}mv^2$

(ii) $\frac{1}{2}mv^2 + ma$

Ans : $K.E = \frac{1}{2}mv^2$

Dimension of K.E = $[ML^2T^{-2}]$

(i) Dimension of $\frac{3}{16}mv^2 = [ML^2\ T^{-2}]$

Since dimensions of K.E

= dimensions of $\frac{3}{16}mv^2$

It is dimensionally correct.

(ii) $\frac{1}{2}mv^2 + ma$

Dimensions = $[ML^2T^{-2}] + [MLT^{-2}]$

which is dimensionally incorrect.

9 Give the dimensional formula for surface energy, moment of inertia, angular velocity and gravitational force. **2**

Ans : Dimensional formula for :

Surface energy = $[ML^2\ T^{-2}]$

Moment of inertia = $[ML^2]$

Angular velocity = $[T^{-1}]$

Gravitational force = $[MLT^{-2}]$

10 It is claimed that two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about 0.02 s. What does this imply for the accuracy of the standard cesium clock in measuring a time interval of 1 s ? **2**

Ans :

100 years = 100 × 365 × 24 × 60 × 60 s = 3.15 × 109 s

In 3.15 × 109 s, the two cesium clocks show a time difference of 0.2 s. Therefore, in 1s the difference in the time shown by two clocks is $\frac{0.2}{3.15 \times 10^9 s} = 0.635 \times 10^{-10}$

Hence, the degree of accuracy in measuring a time interval of 1s is 1 part in $\frac{1}{0.635 \times 10^{-10}}$ or roughly 1 part in 10^{10}.

11 The wavelength l associated with a moving particle depends upon its mass m, its velocity v and Planck's constant h. Show dimensionally the relationship between them. **2**

Ans :

$$\lambda = kh^a m^b v^c$$
$$[M^0L^1T^0] = [ML^2T^{-1}]^a\ [M^b][LT^{-1}]^c$$
$$= M^{a+b}L^{2a+c}T^{-a-c}$$

Applying principle of homogeneity of dimensions

$$a + b = 0$$
$$2a + c = 1$$
$$-a - c = 0$$

Solving the three equations, we get

$a = 1,\ b = -1,\ c = -1.$

$$\lambda = h^1\ m^{-1}v^{-1}$$
$$\lambda = \frac{h}{mv}$$

12 When white light travels through glass the refractive index **2**

$\mu = \left(\frac{\text{Velocity of light in air}}{\text{Velocity of light in glass}}\right)$ is found to vary with wavelength as

$\mu = A + \frac{B}{\lambda^2}$, where A and B are constants. Using the principle of homogeneity of dimensions, determine the SI unit in which A and B must be expressed.

Ans : A is a constant and have no unit and SI unit of B is m^2.

13 Out of formulae (i) $y = a \sin 2\pi t/T$ and (ii) $y = a \sin vt$ for the displacement y of a particle undergoing a certain periodic motion, rule out the wrong formula on dimensional grounds. **2**

[where a = maximum displacement of the particle, v = speed of the particle, T = time period of motion.]

Ans : (i) $\frac{2\pi t}{T}$ have dimensions $M^0L^0T^0$

$\therefore\ y = a$ Hence correct.

(ii) vt have dimensions $M^0L^1T^0$ not dimensionless, so it is wrong.

14 If Length, Time and Energy are fundamental units, find the dimension of mass. **2**

Ans : $K.E. = E = \frac{1}{2}mv^2$

$$[M] = \left[\frac{E}{v^2}\right] = [EL^{-2}T^{+2}]$$
$$= [ML^2T^{-2}L^{-2}T^2] = [M]$$

15 The photograph of a house occupies an area of 1.75 cm^2 on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is 1.55 m^2. What is the linear magnification of the projector-screen arrangement. **2**

Ans : Aerial magnification $(m) = \frac{A_{Image}}{A_{Object}} = \frac{1.55\ m^2}{1.75\ cm^2}$

$\therefore \quad m = 0.88 \times 10^4$

Linear magnification $= \sqrt{m} = 94.11$

16 The radius of a solid sphere is measured to be 11.24 cm. What is the surface area of the sphere to appropriate significant figures ? **2**

Ans : $r = 11.24$ cm

Surface area $= 4\pi r^2 = 4\pi(11.24)^2$

$= 4 \times \frac{22}{7}(11.24)^2$

$= 1588\ cm^2$

17 If $x = at + bt^2$, where x is in metre and t in hour, what will be the unit of 'a' and 'b' ? **2**

Ans : $x = at + bt^2$

So, the units of $a = \frac{x}{t}$ = m/hr

and $b = \frac{x}{t^2} = m/(hr)^2$

18 The length and breadth of a rectangle are (5.7 ± 0.1) cm and (3.4 ± 0.2) cm. Calculate area of the rectangle with error limits. **2**

Ans :

$l = 5.7 \pm 0.1$ cm

$b = 3.4 \pm 0.2$ cm

Area $= lb = 5.7 \times 3.4 = 19.38\ cm^2 = 19.4\ cm^2$

$\Delta A = \left(\frac{\Delta l}{l} + \frac{\Delta b}{b}\right) \times A$

$= \left(\frac{0.1}{5.7} + \frac{0.2}{3.4}\right) \times 19.4$

$= (0.017 + 0.059) \times 19.4$

$= 1.47 \simeq 1.5$

So, $A = (19.4 \pm 1.5)\ cm^2$

19 If A = (12.0 ± 0.1) cm and B = (8.5 ± 0.5) cm, find : (i) A + B (ii) A – B. **2**

Ans :(i) A + B = 20.5 ± 0.6 cm

(ii) A – B = 3.5 ± 0.6 cm

20 Magnitude of force F experienced by a certain object moving with speed v is given by $F = Kv^2$ where K is a constant. Find the dimensions of K. **2**

Ans :

$$[F] = MLT^{-2}$$
$$[v] = LT^{-1}$$
$$F = Kv^2$$
$$[MLT^{-2}] = K[LT^{-1}]^2$$
$$= K[L^2T^{-2}]$$
$$[K] = \frac{[MLT^{-2}]}{[L^2T^{-2}]} = [ML^{-1}]$$

21 For the determination of ‘g’ using a simple pendulum, measurements of l and T are required. Error in the measurement of which of these will have larger effect on the value of ‘g’ thus obtained and why ? What is done to minimize this error ? **2**

Ans :

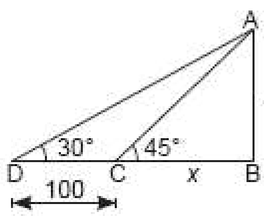
Error in measurement of time period T has larger effect on the value of *g*.

Since $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow g = \frac{l}{T^2}$

Thus, time for large number of oscillations is measured to minimise error.

22 Find the height of a rock mountain, if the angle of elevation of its top increases from 30° to 45° on moving 100 m towards the rock in the horizontal direction through the base of the rock. **2**

Ans :



In ΔABC, $\frac{h}{x} = \tan 45° \Rightarrow h = x$

In ΔABD, $\frac{h}{100+x} = \tan 30°$

$$\frac{h}{100+h} = \frac{1}{\sqrt{3}}$$
$$\sqrt{3}\ h = 100 + h$$
$$h = \frac{100}{\sqrt{3}-1} = 50\left(\sqrt{3}+1\right) = 136.5 \text{ m}$$

23 Percentage error in the measurement of height and radius of cylinder are *x* and *y* respectively. Find percentage error in the measurement of volume. Which of the two measurements height or radius need more attention ? **2**

Ans :

Height of cylinder = x

Radius of cylinder = y

Volume of cylinder $V = \pi y^2 x$

Percentage error in measurement of volume $\frac{\Delta V}{V} \times 100 = \pm\left(2\frac{\Delta y}{y} + \frac{\Delta x}{x}\right) \times 100$

Hence, radius needs more attention because any error in its measurement is multiplied two times.

24 The length and breadth of a rectangle are measured as $(a \pm \Delta a)$ and $(b \pm \Delta b)$ respectively. Find (i) relative error, (ii) absolute error in the measurement of area. **2**

Ans :

(i) Relative error in area $\frac{\Delta A}{A} = \left[\frac{\Delta a}{a} + \frac{\Delta b}{b}\right]$ (ii) Absolute error in area as $A = ab$

$$\Delta A = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)A$$

$$= \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)ab$$

$$\Delta A = [(\Delta a)b + (\Delta b)a]$$

25 Using the principle of homogeneity of dimensions find which of the following is correct. **2**

$(i)\ T^2 = 4\pi^2 r^2$

$(ii)\ T^2 = \frac{4\pi^2 r^3}{G}$

$(iii)\ T^2 = \frac{4\pi^2 r^3}{GM}$

where T is the time period, G is gravitational constant, M is mass and r is radius of orbit.

Ans :

$$[T^2] = T^2$$

$$\left[\frac{4\pi^2 r^3}{GM}\right] = \frac{L^3 M^2}{M^2 L T^{-2} L^2} = T^2$$

So, (iii) is correct.

26 The mean value of period of oscillation of a simple pendulum in an experiment is 2.62 s. The arithmetic mean of all the absolute errors is 0.11 s. Round off the period of simple pendulum to appropriate number of significant figures with reasons. **2**

Ans :

$T = 2.62$ sec. $\overline{T} = 0.11$ sec

$\Rightarrow \therefore T = 2.62 \pm 0.11$ sec.

Error is 0.11, which is 1/10th

Hence T = 2.6 seconds uncertain digit is the '6'.

So, with significance figures we write T = 2.6 seconds

27 The moon is observed from two diametrically opposite points A and B on Earth. The angle θ subtended at the moon by the two directions of observation is 1°54′. Given the diameter of earth to be about 1.276×10^7 m, compute the distance of the moon from the earth. **2**

Ans :

$\theta = 1°54' = 114'$

$= (114 \times 60)'' \times (4.85 \times 10^{-6})$ radian

$= 3.32 \times 10^{-2}$ radian

(since $1'' = 4.85 \times 10^{-6}$ rad.)

$b = 1.276 \times 10^7$ m

$\therefore D = \frac{b}{\theta} = \frac{1.276 \times 10^7}{3.32 \times 10^{-2}}$

$= 3.84 \times 10^8$ m

28 The Sun's angular diameter is measured to be 1920″. The distance *D* of the Sun from the Earth is 1.496×10^{11} m. What is the diameter of the Sun ? **2**

Ans :

Sun's angular diameter $= 1920''$

$= 1920 \times 4.85 \times 10^{-6}$ rad

$= 9.3 \times 10^{-3}$ radian

Sun's diameter

$d = \alpha D$

$= (9.3 \times 10^{-3}) \times (1.496 \times 10^{11})$

$= 1.39 \times 10^9$ m

29 Each side of a cube is measured to be 7.203 m. What are the total surface area and the volume of the cube to appropriate significant figures ? **2**

Ans : Surface area of the cube $= 6 \times a^2$

$= 6 \times (7.203)^2 = 311.299254 = 311.3$ m^2

Volume of the cube $= a^3 = (7.203)^3$

$= 373.7148\ m^3$

$= 373.7$ m^3

30 Fill in the blanks : **3**

(i) The volume of a cube of side 1 cm is equal to .....m^3.

(ii) The surface area of a solid cylinder of radius 2 cm and height 10 cm is equal to......(mm)^2.

(iii) A vehicle moving with a speed of 18 km/h covers....m in 1 sec.

Ans :(i) Volume = $(1\text{cm})^3 = (10^{-2})^3 = 10^{-6}$ m^3

(ii) Surface area

$= 2\pi r(r + h)$

$= 2 \times \pi \times 20$mm (20mm + 100mm)

$= 4800\pi = 4800 \times 3.14$ (mm)^2

= 15070 (mm)^2

(iii) 18 km/h = 5 ms^{-1}

31 Estimate the average mass density of a sodium atom assuming its size to be about 2.5 Å. (Use the known values of Avogadro's number and the atomic mass of sodium). Compare it with the density of sodium in its crystalline phase : 970 kg m^{-3}. Are the two densities of the same order of magnitude ? If so, why ? **3**

Ans :

$$\text{Volume of sodium atom} = \frac{4}{3}\pi r^3$$
$$= \frac{4\pi \times (1.25 \times 10^{-10})^3}{3}$$
$$= 8.180 \times 10^{-30}\ \text{m}^3$$

Now according to Avogadro's hypothesis, one mole of sodium (which contains 6.023×10^{23} atoms) has a mass of 23 g or 23×10^{-3} kg.

∴ The mass of one atom

$M = \frac{23 \times 10^{-3}}{6.023 \times 10^{23}} = 3.82 \times 10^{-26}$ kg Hence, density of sodium atoms

$$\rho_a = \frac{M}{V} = \frac{3.82 \times 10^{-26}}{3.18 \times 10^{-30}} = 4.7 \times 10^3\ \text{kg m}^{-3}$$

Given that density of solid sodium

$\rho_s = 970$ kgm^{-3} = 0.97 × 103 kg m^{-3}.

Thus we find that ρ_a and ρ_s are of the same order of magnitude. This implies that the sodium atoms in the solid are so closely packed that the interatomic separation is of the same order as the size of a sodium atom.

32 A famous relation in physics relates 'moving mass' m to the 'rest mass' m_0 of a particle in terms of its speed v and the speed of light c. This relation first arose as a consequence of special relativity by Albert Einstein. A boy recalls the relation almost correctly but forgets where to put the constant c. He writes : **3**

$$m = \frac{m_0}{(1 - v^2)^{1/2}}.$$

Guess where to put the missing c.

Ans :

c^2 – square of velocity of light should be put in the denominator of v^2, so that $\frac{v^2}{c^2}$ becomes dimensionless. Therefore the correct relation is $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

33 Write the dimensions of **3**

(i) Linear density

(ii) Power

(iii) Impulse

(iv) Velocity gradient

(v) Mass per unit area

(vi) Kinetic energy

(vii) Angular acceleration

(viii) Couple

(ix) Moment of force

(x) Work done

Ans :(i) $[ML^{-1}T^0]$

(ii) $[ML^2T^{-3}]$

(iii) $[MLT^{-1}]$

(iv) $[M^0L^0T^{-1}]$

(v) $[ML^{-2}T^0]$

(vi) $[M^1L^2T^{-2}]$

(vii) $[M^0L^0T^{-2}]$

(viii) $[ML^2T^{-2}]$

(ix) $[ML^2T^{-2}]$

(x) $[ML^2T^{-2}]$.

34 Write the dimensional formula for the following : **3**

(i) Wein's constant

(ii) Planck's constant

(iii) Specific heat

(iv) Latent heat

(v) R ydberg's constant.

Ans :(i) $[M^0LT^0K]$

(ii) $[ML^2T^{-1}]$

(iii) $[M^0L^2T^{-2}K^{-1}]$

(iv) $[M^0L^2T^{-2}]$

(v) $[M^0L^{-1}T^0]$.

35 Name the physical quantity of the dimension given below : **3**

(i) ML^0T^{-3}

(ii) $ML^{-1}T^{-1}$

(iii) $M^{-1}L^3T^{-2}$

(iv) ML^2T^{-3}

(v) ML^0T^{-2}

(vi) T^{-1}

Ans :(i) Energy intensity

(ii) Coeff. of viscosity

(iii) Gravitational Constant

(iv) Power

(v) Surface tension or force constant or spring factor.

36 Write the dimensions of the following : **3**

(i) Gravitational potential

(ii) Variable force

(iii) Pressure gradient

(iv) Moment of inertia

(v) Buoyant force

(vi) Angular momentum

(vii) Work done by torque

(viii) Moment of momentum

(ix) Moment of force

(x) Pressure energy.

Ans :(i) $[M^0L^2T^{-2}]$

(ii) $[MLT^{-2}]$

(iii) $[ML^{-2}T^{-2}]$

(iv) $[ML^2T^0]$

(v) $[MLT^{-2}]$

(vi) $[ML^2T^{-1}]$

(vii) $[ML^2T^{-2}]$

(viii) $[ML^2T^{-1}]$

(ix) $[ML^2T^{-2}]$

(x) $[ML^2T^{-2}]$

37 If $\left(P+\frac{a}{V^2}\right)\ (V-b)=RT$, where the symbols have their usual meanings, **3**

then $\left(\frac{a}{b}\right)$ has a dimension of.... .

Ans : Energy

38 Force (F) and density (d) are related as $F = \frac{\alpha}{\beta + \sqrt{d}}$ **3**

(i) then the dimensions of α are.............,

(ii) then the dimensions of β are...................

Ans :(i) $[M^{3/2}L^{-1/2}T^{-2}]$

(ii) $[M^{1/2}L^{-3/2}T^{0}]$

39The unit of length convenient on the nuclear scale is a fermi : 1 fermi = 10^{-15} m. Nuclear sizes obey roughly the following empirical relation : $r = r_0A^{1/3}$ where r is the radius of the nucleus, A its mass number and r0 is a constant equal to about 1.2 fermi. Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of sodium nucleus. **3**

Ans :

Radius of a nucleus is given by $r = r_0A^{1/3}$

∴ Volume (V) of a nucleus

$= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(r_0A^{1/3})^3 = \frac{4\pi r_0^3 A}{3}$

where $r_0 = 1.2\ f = 1.2 \times 10^{-15}$ m

Now, the mass of nucleus of mass number A is M = A amu.

∴ Density of nucleus

$\rho_n = \frac{M}{V} = \frac{A \times (1.66 \times 10^{-27})}{[(4\pi r_0^3 A)/3]}$

$= \frac{3 \times 1.66 \times 10^{-27}}{4\pi r_0^3}$ kg/m^3

Thus we find that the nuclear mass density ρ_n is independent of mass number A. i.e., all nuclei have roughly the same mass density. Substituting value of r_0 in above equation, the density of any nucleus or density of a sodium nucleus

$\rho_n = \frac{3 \times 1.66 \times 10^{-27}}{4 \times 3.142 \times (1.2 \times 10^{-15})^3}$

$= 2.3 \times 10^{17}$ kg/m^3

40A man walking briskly in rain (speed v) must slant his umbrella (forward) making an angle θ with the vertical. A student derives the following relation between θ and v : tan θ = v and checks that the relation has a correct limit as v → 0, θ → 0 as expected. Do you think this relation can be correct ? If not, guess the correct relation. **3**

Ans :

The given relation is not correct because L.H.S. has no dimensions. The correct formula should be $\tan\theta = \frac{v}{u}$

where u is the speed with which the rain is falling.

41 A book with many printing errors contains four different formulae for the displacement y of a particle undergoing a certain periodic motion : **3**

(i) $y = a \sin \frac{2\pi t}{T}$

(ii) $y = a \sin vt$

(iii) $y = (a/T) \sin (t/a)$

(iv) $y = (a\sqrt{2}) (\sin 2\pi t/T + \cos 2\pi t/T)$

where a = maximum displacement of the particle, v = speed of the particle, T = time-period of motion. Rule out the wrong formulae on dimensional grounds.

Ans :(i) This equation is correct

(ii) This equation is not correct

L.H.S. = L

R.H.S. = L sin $LT^{-1} \times T$ = L sin L (angle is not dimensionless)

(iii) This equation is not correct, as angle is not dimensionless

(iv) This equation is correct.

42 In an experiment, refractive index of glass was observed to be 1.45, 1.56, 1.54, 1.44, 1.54 and 1.53. Calculate (i) Mean value of refractive index ; (ii) Mean absolute error; (iii) Fractional error ; (iv) Percentage error. Express the result in terms of absolute error and percentage error. **3**

Ans : (i) Mean value of μ

$= \frac{1.45+1.56+1.54+1.44+1.54+1.53}{6}$

$\mu = 1.51$

(ii) Mean absolute error

$= \frac{\text{sum of absolute error}}{6}$

$\Delta\bar{\mu} = \frac{0.06+0.05+0.03+0.07+0.03+0.02}{6}$

$= \frac{0.26}{6} = 0.0433 = 0.04$

(iii) Fractional error $= \frac{\Delta\bar{\mu}}{\bar{\mu}} = \frac{0.04}{1.51}$

$= 0.02649 = 0.03$

(iv) Percentage error $= \frac{\Delta\bar{\mu}}{\bar{\mu}} \times 100 = 3\%$

$\mu = 1.51 \pm 0.04$ in terms of absolute error.

Also, $\mu = 1.51 \pm 3\%$ in terms of % error.

43 State the principle of homogeneity of dimensions. Test the dimensional homogeneity of the following equation : **3**

$$h = h_0 + v_0 t + \frac{1}{2} g t^2.$$

Ans :

Principle of Homogenity – All terms of any physical relation must have the same dimensions.

$[h] = L$

$[h_0] = L$

$[v_0 t] = LT^{-1}\ T = L$ So, the equation is dimensionally correct.

$\left[\frac{1}{2} g t^2\right] = LT^{-2}\ T^2 = L$

44 Rule out or accept the following formulae for kinetic energy on the basis of dimensional arguments. **3**

(i) K = 3/16 mv^2

(ii) K = 1/2 $mv^2 + ma$

(iii) K = 1/2 mv^2

Ans :

Dimension of L.H.S. and R.H.S. are same in (i) and (iii). So they are accepted.

(ii) $K = \frac{1}{2} mv^2 + ma$

and $\left[\frac{1}{2} mv^2\right] \neq [ma]$. So it is wrong.

45 A physical quantity Q is given by **3**

$$Q = \frac{A^2 . B^{\frac{3}{2}}}{C^{+4} D^{\frac{1}{2}}}$$

The percentage error in A, B, C, D are 1%, 2%, 4%, 2% respectively. Find the percentage error in Q.

Ans : $\%$ error in $Q = 2\left(\frac{dA}{A} \times 100\right) + \frac{3}{2}\left(\frac{dB}{B} \times 100\right)$

$$+ 4\left(\frac{dC}{C} \times 100\right) + \frac{1}{2}\left(\frac{dD}{D} \times 100\right)$$

$$= 2 \times 1 + \frac{3}{2} \times 2 + 4 \times 4 + \frac{1}{2} \times 2$$

$$= 2 + 3 + 16 + 1 = 22\%$$

46 Answer the following : **3**

(a) You are given a thread and a metre scale. How will you estimate the diameter of the thread ?

(b) A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale ?

(c) The mean diameter of a thin brass rod is to be measured by vernier callipers.

Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only ?

Ans :

(a) Wrap the thread a number of times on a pencil to form a coil having its turns very close to each other. Measure the length of this thread by placing it on a metre scale.

Thickness (diameter) of the thread $= \frac{l}{m}$

where l is the length and m are number of turns in the coil.

(b) It is not possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale. The accuracy can be increased to a large extent by increasing the number of divisions on the circular scale as the least count of the gauge would become less.

(c) It is due to the reason that random error involved in the mean value of 100 observations is much less than that involved in the mean value of 5 observations.

47 The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m and 2.01 cm respectively. Calculate the surface area and volume of the sheet to correct significant figures. **4**

Ans :

Length (l) = 4.234 m

Breadth (b) = 1.005 m

Thickness (h) = 2.01 cm = 0.0201 m

Surface area of the sheet = $2(lb + bh + lh) = 2\ (4.234 \times 1.005 + 1.005 \times 0.0201 + 4.234 \times 0.0201) = 8.7209478$.

As the lowest significant figure in the given measurement is 3 (that of thickness) the volume and area should be expressed in 3 significant figures only.

∴ Surface area = 8.72 m^2

Volume of the sheet = $l \times b \times h = 4.234 \times 1.005 \times 0.0201 = 0.0855289\ m^3 = 0.086\ m^3$

48 Deduce by the method of dimensions, an expression for the energy of a body executing S.H.M. assuming that the energy of the body depends upon (a) the mass m (b) the frequency ν and (c) the amplitude of vibration a. **4**

Ans :

Let E be the energy of the body executing S.H.M.

$$E \propto m^{\alpha} \nu^{\beta} a^{\gamma}$$

or $E = Km^{\alpha}\nu^{\beta}a^{\gamma}$, where K is constant.

Writing the dimensions of both the, sides we have $[ML^2T^{-2}] = [M]^{\alpha}\ [L]^{\gamma}\ [T^{-1}]^{\beta}$

$[ML^2T^{-2}] = [M^{\alpha}\ L^{\gamma}\ T^{-\beta}]$

Comparing the powers of M, L and T, we get

$\alpha = 1$, $\beta = 2$ and $\gamma = 2$. Putting the values of α, β, γ we have, $E = K\,mv^2a^2$

49 To determine acceleration due to gravity, the time of 20 oscillations of a simple pendulum of length 100 cm was observed to be 40 s. Calculate the value of *g* and maximum percentage error in the measured value of *g*. **4**

Ans :

$$T = 2\pi\sqrt{\frac{l}{g}} \text{ or } g = 4\pi^2 \frac{l}{T^2}$$

Given : $l = 100$ cm, $T = \frac{40}{20} = 2.0$ s Let us now calculate the maximum error

$$\therefore \quad g = 4 \times (3.14)^2 \times \frac{100}{(2.0)^2}$$

$$= 985.9 \text{ cm s}^{-2}$$

$$g = 4\pi^2 \left(\frac{l}{T^2}\right)$$

$$= 4\pi^2 \frac{l}{\left(\frac{t}{20}\right)^2} \left(\text{taking } T = \frac{t}{20}\right)$$ Taking log on both sides, we get

$$\text{or} \quad g = \frac{4\pi^2 l \times (20)^2}{t^2}$$

$$\log g = \log 4 + 2\log \pi + \log l - 2\log t + 2\log 20$$ differentiating both the sides

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} - 2\frac{\Delta t}{t}$$

Given : $l = 100$ cm

$\Delta l = 0.1$ cm (least count of the metre scale)

$t = 40$ s, $\Delta t = 0.1$ s (least count of a stop watch)

$\therefore$ Maximum error in g

$$= \frac{0.1}{100} + 2 \times \frac{0.1}{40}$$

$$= 0.001 + 0.005 = 0.006$$

$$= 0.006 \times 100\% = 0.6\%$$

Here 0.1% is the error in the measurement of length, and 0.5% is the error in the measurement of time. Therefore, time needs more careful measurement.

50 By using the method of dimension, check the accuracy of the following formula : **4**

$$T = \frac{rh\rho g}{2\cos\theta},$$

where T is the surface tension, h is the height of the liquid in a capillary tube, ρ is the density of the liquid, g is the acceleration due to gravity, θ is the angle of contact, and r is the radius of the capillary tube.

Ans :

In order to find out the accuracy of the given equation we shall compare the dimensions of T and $\frac{rh\rho g}{2\cos\theta}$

$$T = \frac{\text{force}}{\text{length}} = \frac{[MLT^{-2}]}{[L]}$$

Dimensions of surface tension, Dimension of $\frac{rh\rho g}{2\cos\theta} = [L][L][ML^{-3}][LT^{-2}]$

$$= [MT^{-2}]$$

(2 cos θ has no dimension)

The dimensions of both the sides are the same and hence the equation is correct.

51 The radius of curvature of a concave mirror measured by spherometer is given by $R = \frac{l^2}{6h} + \frac{h}{2}$. **4**

The values of l and h are 4 cm and 0.065 cm respectively. Compute the error in measurement of radius of curvature.

Ans :

Here l = 4 cm, Δl = 0.1 cm

(least count of the metre scale)

here l is the distance between the legs of the spherometer.

h = 0.065 cm, Δh = 0.001 cm = (least count of the spherometer)

As $$R = \frac{l^2}{6h} + \frac{h}{2}$$

$$\frac{\Delta R}{R} = \frac{2\Delta l}{l} + \left(-\frac{\Delta h}{h}\right) + \frac{\Delta h}{h}$$

Considering the magnitudes only, we get

$$\frac{\Delta R}{R} = 2\frac{\Delta l}{l} + \frac{\Delta h}{h} + \frac{\Delta h}{h}$$

$$= 2\left(\frac{\Delta l}{l} + \frac{\Delta h}{h}\right)$$

$$= 2 \times \frac{0.1}{4} + \frac{2 \times 0.001}{0.065}$$

$$= 0.05 + 0.03 = 0.08\%$$

52A physical quantity P is related to four observables, a, b, c and d as follows : **4**

$$P = \frac{a^3 b^2}{\sqrt{c\,d}}$$

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2% respectively. What is the percentage error in the quantity P ? If the value of P calculated using the given relation turns out to be 3.763, to what value should the result be rounded off ?

Ans :

Given : $P = \frac{a^3 b^2}{d\sqrt{c}}$

Percentage error in P is given by,

$$\frac{\Delta P}{P} = 3\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + \frac{\Delta d}{d} + \frac{1}{2}\frac{\Delta c}{c}$$

$$\frac{\Delta P}{P} = 3 \times 1\% + 2 \times 3\% + 1 \times 2\% + \frac{1}{2} \times 4\%$$

$$\left(\because \frac{\Delta a}{a} = 1\%, \frac{\Delta b}{b} = 3\%, \frac{\Delta c}{c} = 4\% \text{ and } \frac{\Delta d}{d} = 2\%\right)$$

$$\Rightarrow \quad \frac{\Delta P}{P} = 3\% + 6\% + 2\% + 2\%$$

$$= 13\%$$

$\therefore$ percentage error in P = 13%

If the calculated number of P is 3.763, the round off value is 3.8.

53 The measured value of length, breadth and height of a block of wood along with maximum permissible errors are expressed as follows : **4**

l = 12.08 ± 0.01 cm, b = 10.12 ± 0.01 cm, h = 5.62 ± 0.01 cm.

Calculate the percentage error in the volume of the block.

Ans : Given : $l \pm \Delta l = 12.08 \pm 0.01$ cm

$b \pm \Delta b = 10.12 \pm 0.01$ cm

$h \pm \Delta h = 5.62 \pm 0.01$ cm

Volume of the block (V) = $l \times b \times h$

percentage error in the volume is given by

$$= \frac{\Delta V}{V} = \left(\frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta h}{h}\right) \times 100\%$$

$$= \left(\frac{0.01}{12.08} + \frac{0.01}{10.12} + \frac{0.01}{5.62}\right) \times 100\%$$

$$= (0.000827 + 0.000988 + 0.00178) \times 100\%$$

$$= 0.003585 \times 100\% = 0.3585\%.$$

54 Reynold's number NR(a dimensionless quantity) determines the condition of laminar flow of a viscous liquid through a pipe. NR is a function of the density of the liquid 'r', its average speed is 'v' and the coefficient of viscosity of the liquid is 'h'. If NR is given directly proportional to 'd' (the diameter of the pipe), show from dimensional consideration that NR $\propto \frac{d\rho v}{\eta}$ the unit of 'η' in SI system is kg m–1 s–1. **4**

Ans :

As the Reynold's Number N_R depends on density ρ, average speed v and coefficient of viscosity η, then let us say

$$N_R \propto \rho^a v^b \eta^c$$

Again N_R is proportional to d, the diameter of the pipe, combining the two we have,

$$N_R \propto \rho^a v^b \eta^c d$$
$$N_R = K \rho^a v^b \eta^c d$$

We write,

$$[N_R] = [M^0L^0T^0]$$
$$[\rho] = [ML^{-3}]$$
$$[v] = [LT^{-1}]$$
$$[\eta] = [ML^{-1}T^{-1}]$$
$$[d] = [L]$$

Substituting the dimension in (i), we have,

$$[M^0L^0T^0] = [ML^{-3}]^a [LT^{-1}]^b [ML^{-1}T^{-1}]^c [L]$$
$$= M^{(a+c)} L^{(-3a+b-c+1)} T^{(-b-c)}$$

Comparing the dimensions of M, L and T, we have,

$$a + c = 0$$
$$-3a + b - c + 1 = 0$$
$$-b - c = 0$$

On simplifying, we get

$$c = -1$$
$$b = 1$$
$$a = 1$$

Therefore, the relation (i) becomes

$$N_R = K\rho^1 v^1 \eta^{-1} d$$
$$N_R = K\rho \frac{vd}{\eta}$$
$$N_R \propto \rho \frac{vd}{\eta}$$

55 A planet moves around the sun in a circular orbit. The time period of revolution T of the planet depends on **4**

(i) R adius of the orbit (R)

(ii) Mass of the sun M

(iii) gravitational constant G

Show dimensionally that $T^2 \propto R^3$

Ans : $T \propto R^a M^b G^c$

or $T = KR^a M^b G^c$, where K is constant

Substituting the dimensions on both sides, we have

$$[T] = [L]^a [M]^b [M^{-1}L^3T^{-2}]^c$$
$$[M^0L^0T] = [M^{b-c} L^{a+3c} T^{-2c}]$$

Comparing the powers of M, L and T, we get

$$b - c = 0$$
$$a + 3c = 0$$
$$-2c = 1$$

On solving, we get $a = \frac{3}{2}$, $b = \frac{-1}{2}$, $c = \frac{-1}{2}$

or $T = KR^{3/2} M^{-1/2} G^{-1/2}$

or $T \propto R^{3/2}$

or $T^2 \propto R^3$

56Experiments show that the frequency (n) of a tuning fork depends upon the length (l) of the prong, the density (d) and the Young's modulus (Y) of its material. From dimensional considerations, find a possible relation for the frequency of the tuning fork. **4**

Ans : Let $n = Kl^a d^b y^c$, where K is constant

Substituting the dimension of all the quantities involved we have : $[T^{-1}] = [L]^a\ [ML^{-3}]^b\ [ML^{-1}T^{-2}]^c$

$$[M^0L^0T^{-1}] = [M]^{b+c}\ [L]^{a-3b-c}\ [T]^{-2c}$$

Comparing powers of M, L and T we get

$$b + c = 0$$

$$a - 3b - c = 0$$

$$-2c = -1$$

or $a = -1$, $b = \frac{-1}{2}$ and $c = \frac{1}{2}$

This gives $n = Kl^{-1}d^{-1/2}y^{1/2}$

or $n = \frac{K}{l}\sqrt{\frac{y}{d}}$

57The density ρ of a piece of metal of a mass m and volume V is given by the formula $\rho = \frac{m}{V}$; If m = 375.32 ± 0.01 g, and V = 136.41 ± 0.01 cm3 Find % error in ρ. **4**

Ans : Given $dm = 0.01$ g and $dV = 0.01$ cm^3

The error in mass,

$$\frac{dm}{m} = \frac{0.01}{375.32}$$

$$= 0.0000266$$

and error in volume

$$\frac{dV}{V} = \frac{0.1}{136.41}$$

$$= 0.0000733$$

As density is a function of both mass and volume the error in its value should be sum of the errors in the mass and volume.

$$\therefore \quad \frac{d\rho}{\rho} = \frac{dm}{m} + \frac{dV}{V}$$

$$= (0.0000266 + 0.0000733)$$

$$= 0.0000999$$

$$\%\text{ error in } \rho = \frac{d\rho}{\rho} \times 100\%$$

$$= 0.0000999 \times 100\%$$

$$= 0.0099\%$$

58It is known that the period T of a magnet of magnetic moment M vibrating in a uniform magnetic field of intensity H depends upon M, H and I where I is the moment of inertia of the magnet about its axis of oscillations. **4**

Show that $T = 2\pi\sqrt{\frac{I}{MH}}$.

Ans :

Let us first write the dimensions of various physical quantities involved, Moment of Inertia,

$M = [M^0L^2A]$ Magnetic moment has the units Am^2, therefore,

$$[H] = \frac{\text{newton}}{\text{A} - \text{m}} = MLT^{-2}L^{-1}A^{-1}$$

or $H = [MT^{-2}A^{-1}]$

Let $T = K\, I^a M^b H^c$.

where K is constant of proportionality,

or, $[T] = [ML^2]^a [L^2A]^b [MT^{-2}A^{-1}]^c$

$= [M^{a+c}\, L^{2a+2b}\, T^{-2c}\, A^{+b-c}]$

Equating powers of M, L and T and A on both the sides, we have,

$a + c = 0$

$2(a + b) = 0$

$\Rightarrow$ $a + b = 0$, $-2c = 1$

and $-c + b = 0$

Solving these equations, we get,

$a = \frac{1}{2},\ b = \frac{-1}{2},\ c = \frac{-1}{2}$

$\therefore$ $T = KI^{1/2}\, M^{-1/2}\, H^{-1/2}$

It can be proved experimentally that

$K = 2\pi$

$\therefore$ $T = 2\pi \sqrt{\frac{I}{MH}}$

59Given that the amplitude of the scattered light is (i) directly proportional to that of incident light, (ii) directly proportional to the volume of the scattering dust particle (iii) inversely proportional to its distance from the scattering particle, and (iv) dependent upon the wavelength (λ) of the light. Show that the intensity of scattered light varies as $\frac{1}{\lambda^4}$. **4**

Ans : Let us write $A_s = K\, A_i^a\, V^b\, x^c\, \lambda^d$

It is clear from the statement of the equation

$a = 1$

$b = 1$

$c = -1$

$\therefore$ $A_s = KA^1_i\, V^1 x^{-1}\, \lambda^d$

Write the dimensions on both the sides and equating powers of M, L and T, we have

$[L] = [L]\ [L^3]\ [L^{-1}]\ [L^d]$

$= [L^{3+d}]$

or $1 = 3 + d$ or $d = -2$

i.e. $A_s \propto \frac{1}{\lambda^2}$

But intensity $(I_s) \propto$ amplitude A_s^2

$\therefore I_s \propto \frac{1}{\lambda^4}$

60Show dimensionally that the frequency n of transverse waves in a string of length l and mass per unit length m under a tension T is given by $n = \frac{K}{l}\sqrt{\frac{T}{m}}$. **4**

Ans : Let us suppose that

$$n = Kl^a T^b m^c$$

where K is a dimensionless constant and a, b, c are unknown powers to be found. Writing the dimension of all the quantities involved, we get

$$[T^{-1}] = [L]^a\ [MLT^{-2}]^b\ [ML^{-1}]^c$$

or $$[T^{-1}] = L^{a+b-c}\ M^{b+c}\ T^{-2b}$$

Comparing powers L, M and T on both sides we have

$a + b - c = 0$, $b + c = 0$ and $-2b = -1$

On simplifying, we get

$$b = 1/2,\ c = -1/2,\ a = -1$$

$\therefore$ $$n = Kl^{-1}\ T^{1/2}\ m^{-1/2}$$

or $$n = \frac{K}{l}\sqrt{\frac{T}{m}}$$

61 If the velocity of light (c), the constant of gravitation (G) and Planck's constant (h) be chosen as the fundamental units, find the dimensions of mass, length and time in the new system. **4**

Ans :

Let us write the dimensions of c, G and h in terms of M, L and T.

$$[c] = [LT^{-1}]$$

$$[G] = [M^{-1}\ L^3T^{-2}]$$

$$[h] = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2\ T^{-1}]$$

Let $M = Kc^\alpha G^\beta h^\gamma$, where K is constant

or $$[M] = [LT^{-1}]^\alpha[M^{-1}L^3T^{-2}]^\beta[ML^2T^{-1}]^\gamma$$

$$= [M^{-\beta+\gamma}\ L^{\alpha+3\beta+2\gamma}\ T^{-\alpha-2\beta-\gamma}]$$

Comparing the powers of M, L and T on both the sides, we have

$-\beta + \gamma = 1$ ...(*i*)

$\alpha + 3\beta + 2\gamma = 0$ ...(*ii*)

$-\alpha - 2\beta - \gamma = 0$ ...(*iii*)

Solving these equations, we get,

$$\alpha = \frac{1}{2},\ \beta = \frac{-1}{2},\ \gamma = \frac{1}{2}$$

$\therefore$ $$M = Kc^{1/2}\ G^{-1/2}\ h^{1/2}$$

Taking K = 1, we can write

$$M = c^{1/2}\ G^{-1/2}\ h^{1/2}$$

Similarly, we can prove that

$$L = c^{-3/2}\ G^{1/2}\ h^{1/2}$$

and $$T = c^{-5/2}\ h^{1/2}\ G^{1/2}$$

62 If the length and time period of an oscillating pendulum have errors of 1% and 2% respectively, what is the error in the estimate of g ? **4**

Ans : We know $T = 2\pi\sqrt{\frac{l}{g}}$ $\therefore$ $g = 4\pi^2 \frac{l}{T^2}$

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

% error in $g = 1 \times 1 + 2 \times 2 = 5\%$

63 The force experienced by a mass moving with a uniform speed v in a circular path of radius r experiences a force which depends on its mass, speed and radius. Prove that the relation is $f = \frac{mv^2}{r}$. **4**

Ans : $f \propto m^a v^b r^c$

$$[f] = [m]^a[v]^b[r]^c$$

$$MLT^{-2} = M^a[LT^{-1}]^b L^c$$

Equating the powers of

$M : 1 = a$

$L : 1 = b + c$

$T : -2 = -b$

Solve the equations to get $a = 1$, $b = 2$, $c = -1$. Therefore, $f = \frac{mv^2}{r}$ is the correct relation.

64 A large fluid star oscillates in space under the influence of its own gravitational field. Using dimensional analysis find the expression for its period (T) of oscillation in terms of radius of star (R) mean density of fluid (ρ) and universal gravitational constant (G). **4**

Ans : Time period $T \propto R^a \rho^b G^c$

$\therefore$ $M^0L^0T^1 = KL^a(ML^{-3})^b(M^{-1}L^3T^{-2})^c$

Equating powers of

$M : 0 = b - c \Rightarrow b = c$

$L : 0 = a - 3b + 3c$

$T : 1 = -2c \Rightarrow c = \frac{-1}{2}$

$\therefore$ $b = c \Rightarrow b = \frac{-1}{2}$

Substitute b and c in

$0 = a - 3b + 3c$

$\therefore$ From (ii), $a = 3b - 3c$

$$a = 3 \times \left(\frac{-1}{2}\right) - 3\left(\frac{-1}{2}\right) = 0$$

$$T = K\sqrt{\frac{G}{\rho}}$$

65 The viscous force 'F' acting on a body of radius 'r' moving with a velocity 'v' in a medium of coefficient of viscosity 'η' is given by F = 6 πηrv. Check the correctness of the formula. **4**

Ans : $F = 6\ \pi\eta r v$

$$[F] = MLT^{-2} \quad ...(i)$$

$$[r]\ [\eta]\ [v] = L.ML^{-1}T^{-1}.LT^{-1}$$

$$= MLT^{-2} \quad ...(ii)$$

Since (i) and (ii) are equal. So, the equation is correct.

66 The number of particles crossing per unit area perpendicular to x-axis in unit time N is given by $N = -D\left(\frac{n_2 - n_1}{x_2 - x_1}\right)$, where n_1 and n_2 are the number of particles per unit volume at x_1 and x_2 respectively. Deduce the dimensional formula for D. **4**

Ans : $D = -N\left(\frac{x_2 - x_1}{n_2 - n_1}\right)$ $\quad [N] = L^{-2}T^{-1}$

$$[D] = \frac{L^{-2}T^{-1}L}{L^{-3}} = L^2T^{-1}, \quad [x_2] = [x_1] = L$$

$$[n_2] = [n_1]$$

$$= \frac{N_0}{L^3} = L^{-3}$$

67 Assuming that the mass (m) of the largest stone that can be moved by a flowing river depends only upon the velocity v, the density ρ of water and the acceleration due to gravity g. Show that m varies, with the sixth power of the velocity of the flow. **4**

Ans :

Let $m \propto v^a \rho^b g^c$

$\therefore m = kv^a\rho^b g^c$, where k is constant

Taking the dimensions of various physical quantities on both the sides, we have,

$$[M] = [LT^{-1}]^a[ML^{-3}]^b[LT^{-2}]^c$$

$$= [M^bL^{a-3b+c}\ T^{-a-2c}]$$

Comparing the powers of M, L and T on both the sides, we have

$$b = 1 \quad ...(i)$$

$$a - 3b + c = 0 \quad ...(ii)$$

$$-a - 2c = 0 \quad ...(iii)$$

Solving these equations we get,

$$b = 1,\ a = 6 \text{ and } c = -3$$

$$\therefore \quad m = kv^6\rho^1 g^{-3}$$

$$m \propto v^6$$

Thus, the mass (m) varies as the sixth power of velocity (v) of the flow.

68 The time of oscillation (t) of a small drop of liquid under surface tension depends upon the density ρ, radius r and surface tension (σ). **4**

Prove dimensionally that $t \propto \sqrt{\frac{\rho r^3}{\sigma}}$.

Ans :

Time of oscillation $t \propto \rho^a r^b \sigma^c$

$\therefore t = k\rho^a r^b \sigma^c$, where k = constant

Writing dimensions of both the sides, we have,

$$[T] = [ML^{-3}]^a \ [L]^b \ [MT^{-2}]^c$$
$$= [M^{a+c} L^{-3a+b} T^{-2c}]$$

Comparing the powers of M, L and T on both sides, we have,

$a + c = 0$ ...(i)

$-3a + b = 0$ ...(ii) Solving equations (i), (ii) and (iii) we get,

$-2c = 1$ ...(iii)

$a = \frac{1}{2}, c = -\frac{1}{2}$ and $b = \frac{3}{2}$ Putting these values in

$t = k\rho^a r^b \sigma^c$

We get $t = k\rho^{1/2} r^{3/2} \sigma^{-1/2}$

or $t \propto \sqrt{\frac{\rho r^3}{\sigma}}$

69Liquid is flowing steadily through a pipe. Assume that the volume of the liquid flowing out per second depends on (a) the coefficient of viscosity of the liquid (η) (b) the radius of the pipe (r) and (c) the pressure gradient along the pipe (pressure gradient is drop in pressure per unit length of the pipe, and is equal to P/l, where P is the difference between the ends of the pipe and l is the length of the pipe). The dimensions of viscosity is $[ML^{-1}T^{-1}]$. Deduce by the method of dimensions, the formula for the volume of the liquid flowing out per second. **4**

Ans :

Let V be the volume of liquid flowing out steadily through the pipe per second

Given : $V \propto \eta^a r^b \left(\frac{P}{l}\right)^c$ where k is constant

Writing the dimensions on both the sides,

or $V = k\eta^a r^b \left(\frac{P}{l}\right)^c$

$[M^0L^3 T^{-1}] = [ML^{-1}T^{-1}]^a \ [L^b] \ [ML^{-2}T^{-2}]^c$

or $[M^0L^3 T^{-1}] = [M^{a+c} L^{-a+b-2c} T^{-a-2c}]$

Comparing the values of M, L and T, we have

$a + c = 0$ ...(i)

$-a + b - 2c = 3$ ...(ii)

$-a - 2c = -1$ ...(iii)

Solving these equations, we get

a = −1, b = 4 and c = 1

Putting these values in $V = k\eta^a r^b \ (P/l)^c$, we have $V = k\eta^{-1} r^4 \left(\frac{P}{l}\right)$

Thus, the volume of the liquid flowing out per second $\frac{kPr^4}{\eta l}$

70 The factors affecting the time period of a simple pendulum are mass, length and the acceleration due to gravity. Deduce a relation for the time period of a simple pendulum. **4**

Ans :

In order to find the correct relationship, suppose the time period T varies as m^a, l^b and g^c, where a, b and c are the powers of m, l and g respectively, i.e., $T \propto m^a l^b g^c = km^a l^b gc$, where k is constant.

Writing the dimensions of both the sides, we have

$$[T] = [M]^a L^b [LT^{-2}]^c$$

or $$[M^0L^0T] = [M^aL^{b+c}T^{-2c}]$$

Comparing the dimensions of M, L and T, we get

$$a = 0$$
$$b + c = 0$$
$$-2c = 1$$

or $$a = 0,\ b = \frac{1}{2} \text{ and } c = \frac{-1}{2}$$

Substituting the values of a, b, c, the required expression of the time period of the pendulum is

$$T = kM^0L^{1/2}g^{-1/2}$$

or $$T = k\sqrt{\frac{l}{g}}$$

71 If two resistances of values $R_1 = (2.0 \pm 0.1)\ \Omega$ and $R_2 = (12.3 \pm 0.2)\ \Omega$ are put (i) in parallel and (ii) in series, find the error in the equivalent resistance. **5**

Ans : (i)

$$R_1 = 2,\ \Delta R_1 = 0.1,$$
$$R_S = R_1 + R_2 = 14.3$$
$$R_2 = 12.3,\ \Delta R_2 = 0.2,$$
$$\Delta(R_1 + R_2) = \Delta R_S = 0.3$$
$$\text{In series } R_S = (R_1 + R_2) \pm \Delta(R_1 + R_2)$$
$$= (14.3 \pm 0.3)\ \Omega$$

(ii) In parallel, $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1+R_2}{R_1R_2}$$

$$\therefore \quad R = \frac{R_1R_2}{R_1+R_2} = \frac{R_1R_2}{R_S}$$

$$\frac{\Delta R}{R} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta R_S}{R_S}$$

$$\Delta R = R\left[\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} + \frac{\Delta R_S}{R_S}\right]$$

$$\Rightarrow \quad R = \frac{24.6}{14.3} = 1.72$$

$$\Delta R = R\left(\frac{0.1}{2} + \frac{0.2}{12.3} + \frac{0.3}{14.3}\right)$$
$$= R(0.05 + 0.016 + 0.020)$$
$$= 1.72\ (0.08) = 0.13$$

$$R \text{ (in parallel)} = (1.72 \pm 0.13)\ \Omega$$