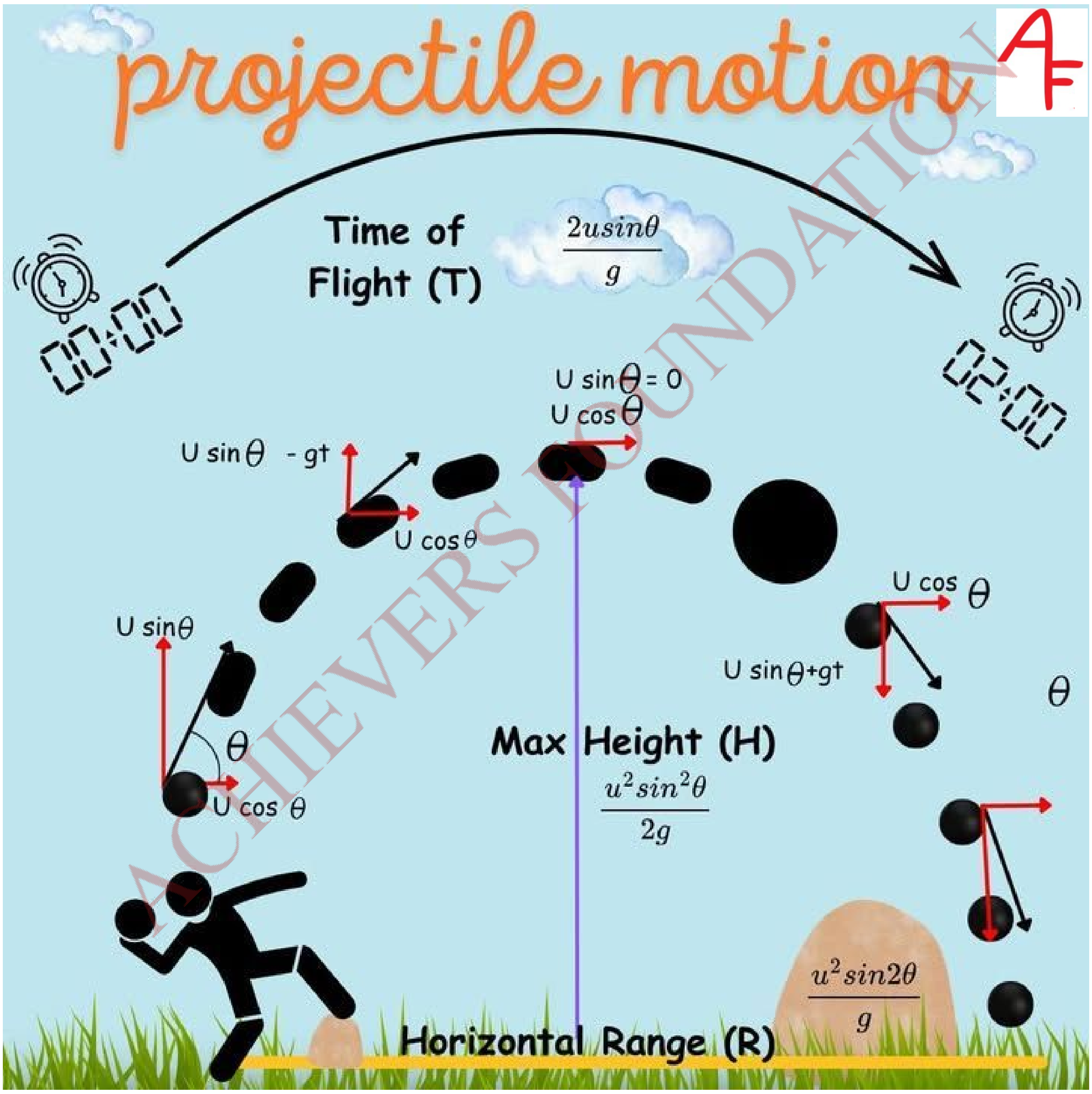
Motion in a Plane



VECTOR ANALYSIS 1

TOPIC 1

SCALAR AND VECTOR QUANTITIES

There are number of physical quantities, but all these are grouped in two categories:

(A) Scalar quantity

(B) Vector quantity

Scalar Quantities

A physical quantity that can be described completely by its magnitude only and does not require a direction is known as a scalar quantity.

It obeys the ordinary rules of algebra.

Example: Distance, Time, Speed, Density, Volume, Temperature, Electric current, etc.

Vector Quantities

A physical quantity that requires magnitude and a particular direction when it is expressed is known as vector quantity.

Vector quantities must obey the rules of vector algebra arrows.

Example: Velocity, acceleration, force, etc.

It is always represented by a line headed with an arrow.

As if $\vec{A}$ is vector

$\vec{A} = \overrightarrow{PQ}$

Magnitude of $\vec{A} = |\vec{A}|$ or A

So, a vector is represented by magnitude and direction, as in figure, point P is called tail and point Q is called head.

So $\vec{A} = \overrightarrow{PQ}$

Types of Vector

Vector quantities are classified into two categories.

(A) Polar vector

(B) Axial vector

Polar Vector

A vector quantity whose direction is along the direction of motion of a body or particle is known as a polar vector.

Example: Displacement, Velocity, Linear momentum, Force, etc.

Suppose a bike is moving eastward and covers 1 km. The displacement is 1 km due east, which means the direction of displacement of the car and its direction of motion, are the same.

Axial Vector

A vector quantity whose direction is along the axis of rotation of the body or particle is called as axial vector.

Example: Angular velocity ($\vec{w}$), angular acceleration ($\vec{a}$), torque ($\vec{\tau}$).

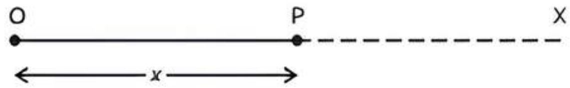
Suppose a top is moving anticlockwise about an axis of rotation under the action of force, then the torque (turning effect of the force) acting on the body is along the axis of rotation.

TOPIC 2

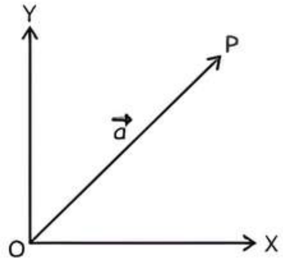
POSITION AND DISPLACEMENT VECTOR

Position Vector

A vector drawn from the origin to the position of a particle at any instant is called a position vector. Consider a particle at point P at a distance from the origin along the x-axis, then the position vector of the particle = $\overrightarrow{OP} = \vec{x}$ as in figure.



Now consider that particle is placed in Plane at time, then $\overrightarrow{OP} = \vec{a}$ which is the position vector of the particle at the instant t.



Displacement vector

The displacement vector of a moving particle in a given interval of time is a directed line segment from the initial to the final position of the particle.

It represents the change in the position of a moving particle in the given time interval.

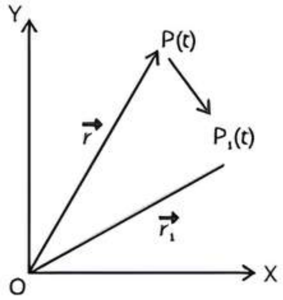
Suppose, a particle is moving in plane, at any instant t, its position is P.

So its position vector is $\vec{OP} = \vec{r}$.

After time t_1', particle moves to the new position P_1

now its position vector is $\vec{OP_1} = \vec{r_1}$

So, the displacement vector of the particle is $\vec{PP_1}$.



Some Important Definitions

Unit Vector: A vector having a magnitude of 1 (unit) is known as a unit vector. It's used to specify the direction and represented as $\hat{P}$.

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

The unit vector in the direction of $\vec{A}$ is,

$$\therefore \quad \text{unit vector} = \frac{\text{vector}}{\text{magnitude of vector}}$$

$$\vec{A} = A\hat{A} = |A|\,\hat{A}$$

Null or zero vector: A vector having zero magnitude.

Important

➥ *A unit vector is used to specify the direction of a vector.*

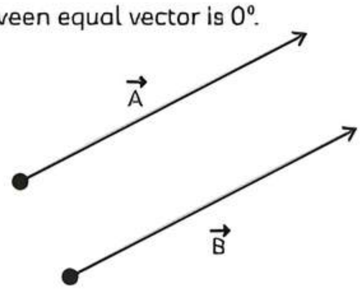
Caution

➥ *Students should know that sum of two vectors is always a vector $\vec{p} + \left(-\vec{p}\right) = \vec{0}$. Here, $\vec{0}$ is a zero vector or null vector.*

Equal vector: These vectors have equal magnitude and same direction.

$$\vec{A} = \vec{B}$$

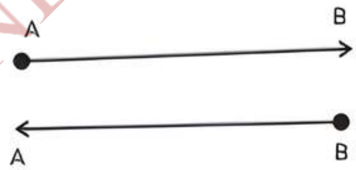
Angle between equal vector is 0°.



Negative vector: These vectors have equal magnitude but opposite direction.

$\vec{AB}$ and $\vec{BA}$ are negative vectors. They have opposite directions are also called antiparallel vectors. The angle between them is 180°.

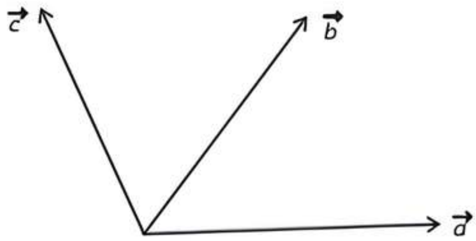
$$\vec{AB} = -\vec{BA}$$



Collinear vectors: Vectors lying in the same line are collinear vectors. Angle between them is 0° or 180°.

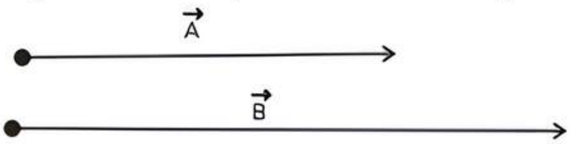
Co-planar vectors: Vectors located in the same plane are called co-planar vectors.

Co-initial vector: Vectors which have the same initial point as shown in figure $\vec{a}, \vec{b}$ and $\vec{c}$ are co-initial vectors.

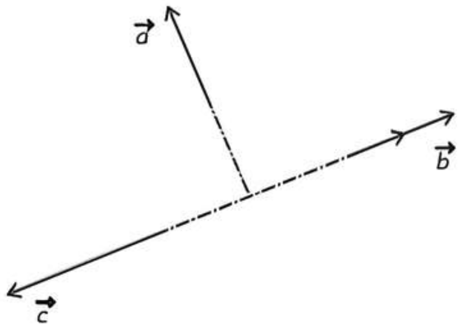


Parallel vector: Vectors which have the same direction are called parallel vectors.

Angle between two parallel vectors is always 0°.



Concurrent vector: Those vectors which pass through a common point are concurrent vectors. In the figure $\vec{a}, \vec{b}$ and $\vec{c}$ are concurrent vectors.



Example 1.1: For satisfying the vectors to be equal, what conditions must exist.

Ans. Two vectors are equal, if they have equal magnitude and are parallel to each other, which means their directions must be the same.

Example 1.2: State whether the following statements are true or false:

(A) The magnitude of a vector is always a scalar.

(B) Each component of a vector is always a scalar.

(C) Three vectors not lying in a plane can never add up to give a null vector.

Ans. (A) True, the magnitude of a vector is a number. So, is it a scalar. Hence, the total path length is always greater than the magnitude of displacement. It becomes equal to the magnitude of displacement only when a particle is moving in a straight line.

(B) False, the vector is the sum of its components as the magnitude of each component is a scalar.

(C) True, the resultant of two vectors must have the same magnitude and opposite direction as that of the third vector to have a null vector.

TOPIC 3

MULTIPLICATION, SUBTRACTION AND ADDITION OF TWO VECTORS

Vector addition can be performed by using the following methods:

(A) Graphical methods.

(B) Analytical methods.

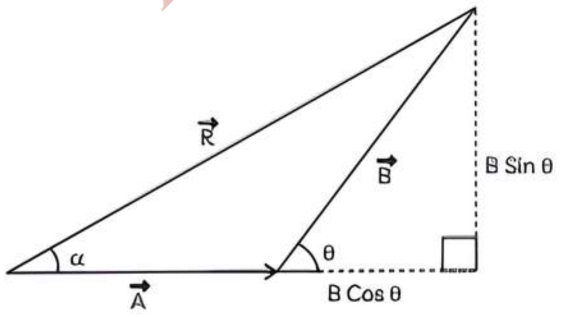
The addition of two vectors is quite different from a simple algebraic sum of two numbers.

Triangle Law of Addition of Two Vectors

If two vectors are represented by two sides of a triangle in the same order, then their sum or resultant vector is given by the third side of the triangle taken in the opposite order of the first two vectors.

Consider a triangle directed from the tail of $\vec{A}$, the head of $\vec{B}$ and resultant $\vec{R}$

The Sum of two vectors is called the resultant vector of these two vectors.



Resultant, $\vec{R} = \vec{A} + \vec{B} = |\vec{A}+\vec{B}|$

Length of $\vec{R}$ is the magnitude of the vector sum.

i.e., ∴ $|\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{(A + B\cos\theta)^2 + (B\sin\theta)^2}$

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Let the direction of $\vec{R}$ make an angle α with $\vec{A}$

∴ $$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

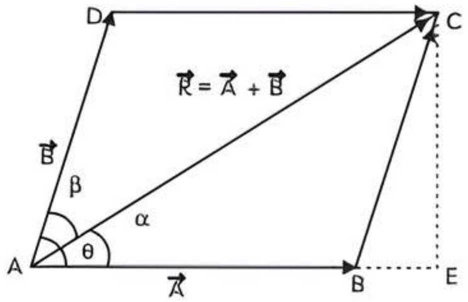
Parallelogram Law of Addition of Two Vectors

If two vectors are represented by two adjacent sides of a parallelogram which are directed away from their common point, then their sum (*i.e.*, resultant vector) is given by the diagonal of the parallelogram paring away through that common point.

$$\vec{AB} + \vec{AD} = \vec{AC}$$

$$\vec{A} + \vec{B} = \vec{R}$$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$



$$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$

Example 1.3: Two forces of magnitude 3 N and 4 N, respectively, are acting on a body. The resultant force, if the angle between them is 0°:

(a) 7 N

(b) 3 N

(c) 0

(d) 4 N

Ans. *(a) 7 N*

Explanation: As $\theta = 0°$, both forces are parallel,

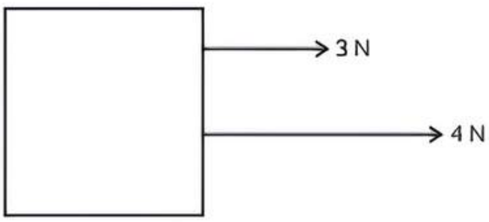
$$R = A + B$$

Net force or resultant force

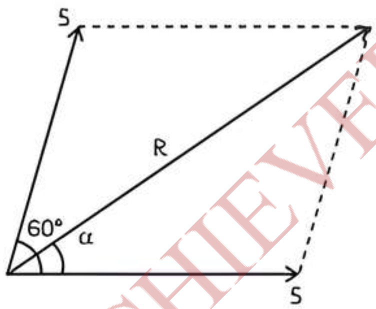
$$R = 3 + 4$$

$$= 7 \text{ N}$$

Direction of resultant is along both force.



Example 1.4: Two vectors have equal magnitude of 5 units, have an angle of 60° between them. Find the magnitude of their resultant vector and its angles from one of the vectors.



Ans. $A = B = 5$ unit and $\theta = 60°$

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$= \sqrt{5^2 + 5^2 + 2 \times 5 \times 5 \cos 60°}$$

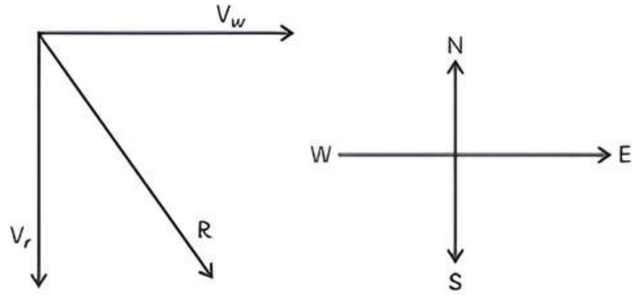
$$= 5\sqrt{3} \text{ unit}$$

$$\therefore \tan \alpha = \frac{B \sin 60°}{A + B \cos 60°} = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{1}{\sqrt{3}}$$

$$= \tan 30°$$

Or $\alpha = 30°$

Example 1.5: In a heavy storm, rain is falling with the speed of after some time the speed 45 ms^{-1} of wind becomes 12 ms^{-1} in the west to east direction. To save himself from the rain in which direction should the boy take his umbrella?



Ans. Let, velocity of rain = v_r

velocity of wind = v_w

According to rule of vector addition, R represent the addition and its magnitude is:

$$R = \sqrt{v_r^2 + v_w^2} = \sqrt{(45)^2 + (12)^2}$$

$$= 46.57 \text{ ms}^{-1}$$

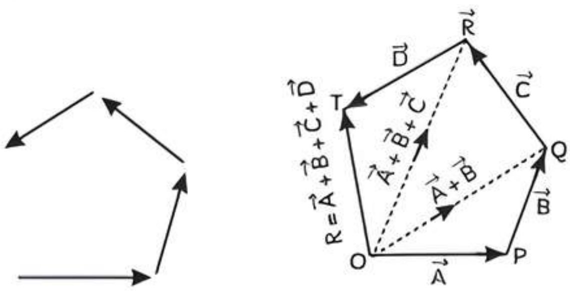
and angle is, $\tan \theta = \frac{v_m}{v_r} = \frac{12}{45} = 0.266$

or, $\theta = \tan^{-1}(0.266) = 14.89°$

Therefore, the boy holds the umbrella in vertical plane at an angle about 15° (approx) with the vertical towards the west.

Polygon Law of Vector Addition

When the number of vectors are represented in both magnitude and direction by sides of an open polygon taken in an order, then their resultant is represented in both magnitude and direction by the closing side of a polygon taken in opposite order.



(a) (b)

Thus, $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

Subtraction of Two Vectors

If $\vec{A}$ and $\vec{B}$ are two vectors, their difference $\vec{A} - \vec{B}$ and be treated as sum of vector $\vec{A}$ and $\left(-\vec{B}\right)$ vector i.e.

$$\vec{A} - \vec{B} = \vec{A} + \left(-\vec{B}\right)$$

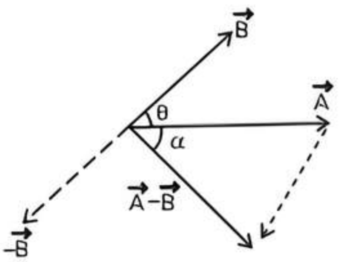
According to law of triangle

$$\vec{A}-\vec{B} = \sqrt{A^2+B^2+2AB\cos(\pi-\theta)}$$

$$= \sqrt{A^2+B^2+2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin\theta}{A-B\cos\theta}$$

Where, θ is the angle between $\vec{A}$ and $\vec{B}$.



Important

➥ *Vector subtraction doesn't follow*

(i) *Commutative law* $\vec{A}\ \vec{B} \neq \vec{B}\ \vec{A}$

(ii) *Associative law* $\left(\vec{A}-\vec{B}\right)-\vec{C} \neq \vec{A}-\left(\vec{B}-\vec{C}\right)$

Example 1.6: A ball having linear momentum strikes a rigid wall in a direction normal to its surface and rebounds with the same momentum in a direction normal to the surface of the wall. Calculate the change in momentum of the ball.

Ans. Let $\vec{A}_1 = \vec{A}$.

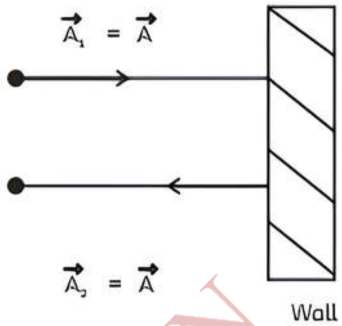
the linear momentum of the ball striking the wall.

Change in momentum

$$\vec{A}_2 = -\vec{A}$$

$$\Delta\vec{A} = -\vec{A}-\vec{A} = -2\vec{A}$$

Negative sign implies that changes in momentum of the ball is in the direction of rebound.



Multiplication of a Vector

By a real number - multiplication of a vector by a number *n* gives a new vector whose magnitude is *n* times the magnitude of the given vector and direction is same as that of the given vector when *n* is a positive real number.

(1) Let $\vec{a}$ the vector be multiplied by a real number '*n*', then the new or resultant vector is, $\vec{A} = n\vec{a}$.

(2) **By a scalar** - when a vector quantity is multiplied by a scalar quantity, then the new physical quantity obtained is also a vector quantity whose magnitude is the product of scalar quantity and vector quantity and direction is the same as that of the vector quantity with which a scalar quantity is multiplied.

TOPIC 4

RESOLUTION OF VECTOR

In plane [two-dimensional]

When a vector is split into components which are at right angles to each other then the components are called rectangular or orthogonal components of the vector.

Let $\vec{A}_x$ and $\vec{A}_y$ be two components of $\vec{A}$, $\hat{i}$ and $\hat{j}$ be the unit vectors along OX and OY respectively, then,

$$\vec{A}_x = A_x\hat{i} \text{ and } \vec{A}_y = A_y\hat{j}$$

$$A_x = \text{magnitude of } A_x\hat{i}$$

$$A_y = \text{magnitude of } \vec{A}_y$$

Then,

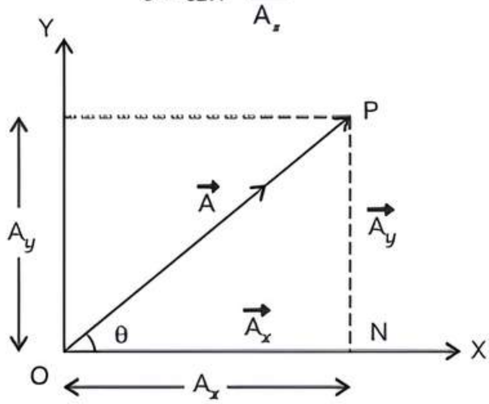
$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

$$A_x = A\cos\theta;\ A_y = A\sin\theta$$

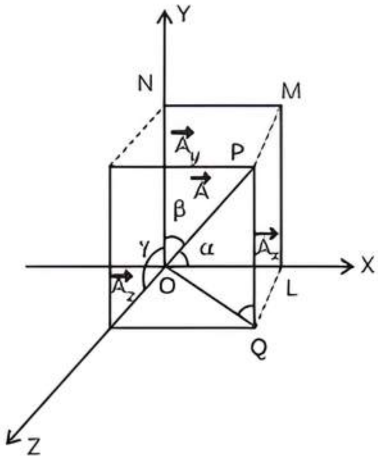
Then magnitude of resultant vector $|A| = \sqrt{A_x^2 + A_y^2}$

If θ is the inclination of A with *x*-axis, then angle θ:

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$



In Three-dimensional



Consider a vector represented by $\vec{A}$ in space. The components of $\vec{OP}$ along OX, OY and OZ are

$\vec{OL} = \vec{A}_x$ $\quad$ $\vec{ON} = \vec{A}_y$ $\quad$ $\vec{OR} = \vec{A}_z$

$\vec{A}_x = A_x\hat{i}$ $\quad$ $\vec{A}_y = A_y\hat{j}$ $\quad$ $\vec{A}_z = A_z\hat{k}$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

Resolution vector,

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Magnitude of vector,

$$\cos\alpha = \frac{A_x}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = l$$

$$\cos\beta = \frac{A_y}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = m$$

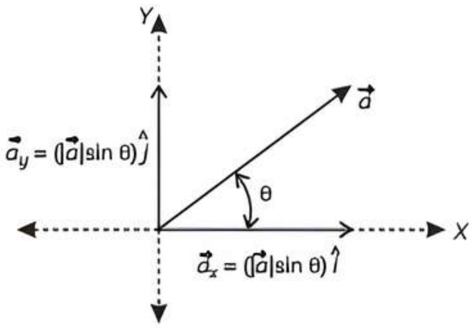
$$\cos\gamma = \frac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = n$$

l, m and n are directions.

$$l^2 + m^2 + n^2 = 1$$

Example 1.8: Case Based:

The resolution of a vector is the splitting of a single vector into two or more vectors in different directions, which together produce a similar effect as is produced by a single vector itself. The vector formed after splitting is called component vectors.



(A) A force of 4 N is indeed at an angle of 60° from the vertical. Find out its components along horizontal and vertical directions.

(B) Determine that vector when added to the resultant of $\vec{P} = 2\hat{i} + 7\hat{j} - 10\hat{k}$ and $\vec{Q} = \hat{i} + 2\hat{j} - 3\hat{k}$ gives a unit vector along x-axis.

(C) The sum of 3 vectors shown in the figure is zero, what is the magnitude of the vector $\vec{OB}$ and $\vec{OC}$?

(a) 5 N
(b) 10 N
(c) 20 N
(d) 15 N

(D) Addition of 3 vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$, which, have an equal magnitude of 50 units and are inclined at angles of 45° and 135° respectively is:

(a) 100 units
(b) 75 unit
(c) 50 unit
(b) 25 unit

(E) A force of 4 N makes an angle of 30° with the x-axis. Find the x and y components of the force.

Ans. (A) Here $\quad F = 4$ N and $\theta = 60°$

So, vertical component,

$$F\cos\theta = 4\cos 60° = 2 \text{ N}$$

Horizontal component,

$$F\sin\theta = 4\sin 60° = 2\sqrt{3} \text{ N}$$

(B) $\vec{P} = 2\hat{i} + 7\hat{j} - 10\hat{k}$ and $\vec{Q} = \hat{i} + 2\hat{j} + 3\hat{k}$

When $\vec{P}$ and $\vec{Q}$ added, it gives the resultant vector is,

Resultant, $\vec{R} = \vec{P} + \vec{Q}$

$$= \left(2\hat{i} + 7\hat{j} - 10\hat{k}\right) + \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

$$= 3\hat{i} + 9\hat{j} - 7\hat{k}$$

Now the obtained vector is along x – axis which means

$$= \hat{i} - \vec{R}$$

$$= \hat{i} - \left(3\hat{i} + 9\hat{j} - 7\hat{k}\right)$$

$$= -2\hat{i} - 9\hat{j} + 7\hat{k}$$

(C) *(b) 10 N*

Explanation: According to the figure, resolving $\vec{OC}$ in two rectangular components.

OD = OC cos 45° and OE = OC sin 45°

For zero resultant

$$OE = OA$$

or $$OC \sin 45° = 10\text{ N}$$

$$OC \times \frac{1}{\sqrt{2}} = 10\text{ N}$$

$$\Rightarrow \quad |\overline{OC}| = 10\sqrt{2}\text{ N}$$

and

$$OD = OB$$

$$\Rightarrow \quad OC \cos 45° = OB$$

$$\Rightarrow \quad 10\sqrt{2} \times \frac{1}{\sqrt{2}} = OB$$

So, $$OB = 10\text{ N}$$

(D) *(c) 50 unit*

Explanation: The angle between $\vec{B}$ and $\vec{C}$ is equal to 315° – 135° = 180°

So, they balance each other.

So, the sum of these three is $\vec{A}$ *i.e.*, 50 units at 45° from *x*-axis.

(E) F = 4 N and θ = 30°

x-component of force,

$$F_x = F\cos\theta$$

$$= 4 \times \cos 30°$$

$$= 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}\text{ N}$$

y-component of force,

$$F_y = F\sin\theta$$

$$= 4 \times \sin 30°$$

$$= 4 \times \frac{1}{2} = 2\text{ N}$$

Important

- *Scalar product or dot product of two vectors*

$$\vec{A}.\vec{B} = AB\cos\theta$$

- *Vector product or cross product of two vectors*

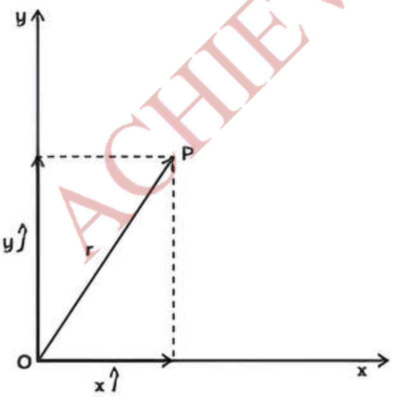
$$\hat{A} = \frac{\vec{A}\times\vec{B}}{AB\sin\theta} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$$

TOPIC 5

POSITION VECTOR AND DISPLACEMENT IN PLANE

The position vector $\vec{r}$ of a particle P, located in a plane with reference to the origin of an *x y*–coordinate system is given by,

$$\vec{r} = x\hat{i} + y\hat{j}$$



Now, if the particle moves along the path as shown to a new position P with the position vector $\vec{r}_1$.

$$\vec{r} = x_1\hat{i} + y_1\hat{j}$$

Change in position of the particle is nothing but its displacement is given by

From the figure, it can also be seen that,

$$\vec{r} + \Delta\vec{r} = \vec{r} \text{ or } \Delta\vec{r} + \vec{r}_1 - \vec{r}.$$

which is nothing but the triangle law of vector addition.

Example 1.9: The position coordinates of a particle in a plane are (1, 1) and (2, 3) at time t_1 and t_2 respectively. What are the position vectors of the particle at t_1 and t_2, and displacement of the particle during this interval of time?

Ans. Position Vector at t_1 is, $\vec{r}_1 = x_1\vec{i} + y_1\vec{j} = \hat{i} + \hat{j}$

Position Vector at t_2 is, $\vec{r}_2 = x_2\vec{i} + y_2\vec{j} = 2\hat{i} + 3\hat{j}$

Displacement of particle

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$= (2\hat{i} + 3\hat{j}) - (\hat{i} + \hat{j}) = \hat{i} + 2\hat{j}$$

The magnitude of displacement of the particle is given by,

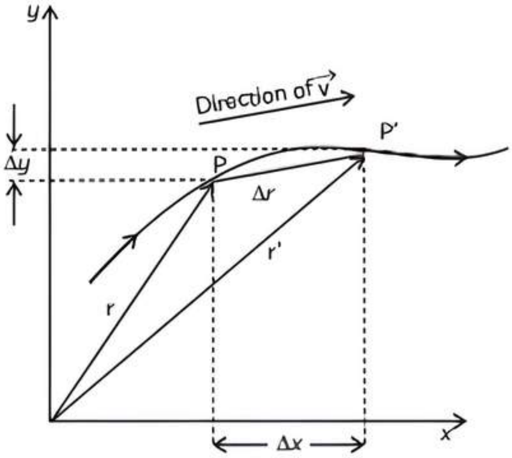
$$|\Delta\vec{r}| = \sqrt{(1)^2 + (2)^2} = \sqrt{5}\text{ unit}$$

TOPIC 6

AVERAGE, INSTANTANEOUS VELOCITY AND ACCELERATION

Average Velocity

The average velocity (v) of an object is the ratio of the displacement and the corresponding time interval.



$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t}$$

$$\vec{v}_{avg} = v_x\hat{i} + v_y\hat{j}$$

Instantaneous Velocity

Instantaneous velocity is given by the limiting value of the average velocity as the time interval approaches zero.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{d\vec{r}}{dt}$$

$\Rightarrow$ $$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

Here, $v_x = \frac{dx}{dt}$ and $v_y = \frac{dx}{dt}$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

Also, $\tan\theta = \frac{v_y}{v_x}$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Average Acceleration

Average acceleration is given by,

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta \vec{v}_x}{\Delta t}\hat{i} + \frac{\Delta \vec{v}_y}{\Delta t}\hat{j}$$

$$\vec{a}_{avg} = a_x\hat{i} + a_y\hat{j}$$

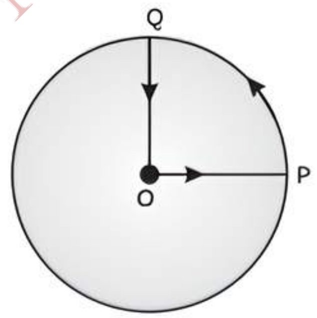
Instantaneous Acceleration

Instantaneous acceleration is given by,

$$\vec{a} = \frac{dv}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$\Rightarrow$ $$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

Example 1.10: **A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference and returns to the centre along OQ as shown in the figure. If the round trip takes ten minutes, calculate the net displacement, average speed and velocity of the cyclist.**



[NCERT]

Ans. Since the initial position coincides with the final position.

So, the net displacement of the cyclist = zero

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}}$$

The net displacement of the cyclist is zero.

so its average velocity is also zero.

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{OP + Arc(PQ) + OQ}{\frac{10}{60}}$$

$$= \frac{r + \frac{2\pi r}{4} + r}{\frac{1}{6}} = \frac{\frac{25}{7}}{\frac{1}{6}} = 21.43 \text{ kmh}^{-1}$$

Example 1.11: A stone, tied to the end of a string of 80 cm long, is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 sec, then what is the magnitude of acceleration of the stone. [NCERT]

Ans. Given, $r = 80$ cm, $v = \dfrac{14}{25}$ rps = 0.56 rps

$$\omega = 2\pi v = 2 \times 3.14 \times 0.56 = 3.52 \text{ rads}^{-1}$$

The acceleration of stone is;

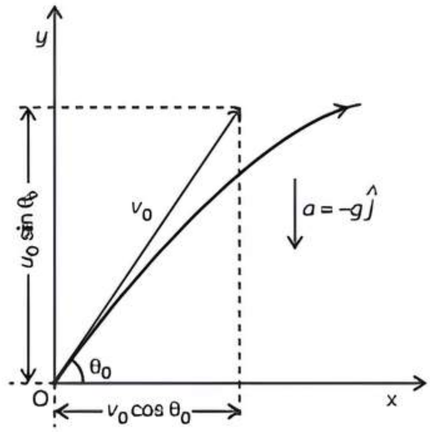
$$a = r\omega^2 = 80 \times 3.52 \times 3.52 = 991.2 \text{ cms}^{-2}$$

The acceleration is directed along the radius of the circular path towards the centre of the circle.

TOPIC 7

PROJECTILE MOTION

When an object is projected obliquely close to the surface of the earth, it moves simultaneously in horizontal and vertical directions. Motion of such an object is referred to as projectile motion.



Here, an object is projected at an angle with an initial velocity 'u'.

Suppose that the projectile is launched with velocity v_0 that makes an angle θ_0 with the x-axis

After the object has been projected, the acceleration acting on it is that due to gravity which is directed vertically downward.

$$a = -g\hat{j}$$

$$a_x = 0, a_y = -g$$

The components of initial velocity v_0 are,

$$v_{ox} = v_0 \cos\theta_0$$

$$v_{oy} = v_0 \sin\theta_0$$

if, $\quad x_0 = 0, y_0 = 0$

Then, $\quad x = v_{ox} t = (v_0 \cos\theta_0)t$

and, $\quad y = (v_0 \sin\theta_0)\, t - \left(\frac{1}{2}\right) g t^2$

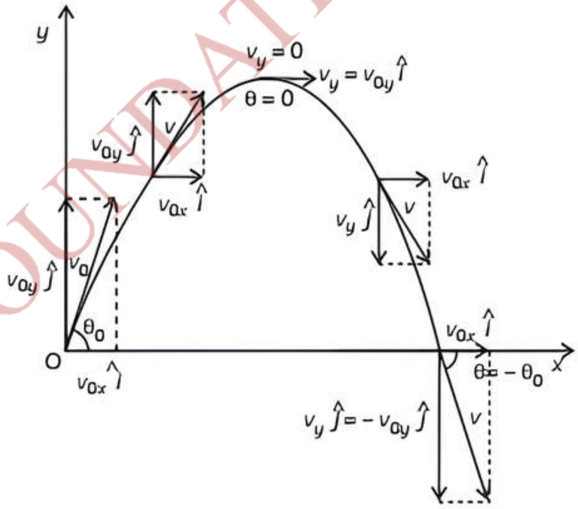
And $\quad \theta = \tan^{-1} \dfrac{v_y}{v_x}$

Equation of Path of a Projectile

The shape of the path followed by the projectile is obtained by eliminating the time between the expressions for x and y in the above equations –

$$y = (\tan\theta_0)x - \frac{g}{2(v_0 \cos\theta_0)^2} x^2$$

Now, since g, θ_0 and v_0 are constants, and equation is in the form of $y = ax + bx^2$, in which a and b are constants. This is the equation of a parabola, *i.e.*, the path of the projectile is a parabola.



Time of Maximum height

The time taken by the projectile to reach the maximum height can be denoted by t_m.

Since, at this point, $v_y = 0$,

$$v_y = v_0 \sin\theta_0 - g t_m = 0$$

Or, $\quad t_m = \dfrac{v_0 \sin\theta_0}{g}$

The total time t_f during which the projectile is in flight can be obtained by putting $y = 0$

$$t_f = \frac{2(v_0 \sin\theta_0)}{g}$$

t_f is known as the time of flight of the projectile.

As $\quad t_f = 2t_m$,

which is expected because of the symmetry of the parabolic path.

Maximum Height of a Projectile

The maximum height h_m reached by the projectile can be calculated by substituting, $t = t_m$ in above equation.

$$y = h_m = (v_0 \sin \theta_0)\left(\frac{v_0 \sin \theta_0}{g}\right) - \frac{g}{2}\left(\frac{v_0 \sin \theta_0}{g}\right)^2$$

Or, $$h_m = \left(\frac{v_0 \sin \theta_0}{2g}\right)$$

Horizontal Range of a Projectile

The horizontal distance travelled by a projectile from its initial position ($x = y = 0$) to the position, where it passes $y = 0$ during its fall is called the horizontal range, R. It is the distance travelled during the time of flight t_f.

Therefore, the range R is,

$$R = (v_0 \cos \theta_0)\,(t_f)$$

$$R = \frac{(v_0 \cos \theta_0)(2v_0 \sin \theta_0)}{g}$$

or, $$R = \frac{v_0^2 \sin 2\theta_0}{g}$$

The equation shows that, for a given projection velocity v_0, R is maximum when $\sin 2\theta_0$ is maximum, *i.e.*, when $\theta_0 = 45°$.

The maximum horizontal range is,

Therefore,

$$R_m = \frac{v_0^2}{g}$$

Important

➥ *At maximum height, angle between velocity and acceleration is 90°.*

Example 1.12: Suppose a particle is moving in x-y plane. Now can there be motion in two directions with an acceleration in only one dimension?

Ans. Yes, the projectile motion is a motion in two dimensions with acceleration in only one dimension *i.e.*, vertical direction only.

Related Theory

➥ *There's no acceleration in the horizontal direction since gravity does not pull projectiles sideways, only downward. Air resistance would cause a horizontal acceleration, slowing the horizontal motion, but since we're only going to consider cases where air resistance is negligible, we can assume that the horizontal velocity is constant for a projectile.*

Example 1.13: Case Based:

A projectile is an object upon which the only force is gravity. Gravity acts to influence the vertical motion of the projectile, thus causing a vertical acceleration. The horizontal motion of the projectile is the result of the tendency of any object in motion to remain in motion at constant velocity. Due to the absence of horizontal forces, a projectile remains in motion with a constant horizontal velocity. Horizontal forces are not required to keep a projectile moving horizontally. The only force acting upon a projectile is gravity.

(A) When a projectile is projected at an angle with the horizontal, then which of the components of the velocity remains constant throughout the trajectory?

(B) Is the speed of a projectile zero at the highest point of its trajectory, when thrown at some angle with the horizontal.

(C) An airplane is flying at a constant height of 1960 m with speed 600 km h^{-1} above the ground towards a point directly over a person struggling in flood water. At what angle of sight with the vertical should the pilot release a survival kit if it is to reach the person in water? ($g = 9.8$ ms^{-2})

(a) 30° (b) 60°

(c) 90° (d) 120°

(D) Assertion (A): When a body is dropped or thrown horizontally from the same height, it would reach the ground at same time.

Reason (R): Horizontal velocity has no effect on the vertical direction.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

[Delhi Gov. QB 2022]

AF

(E) **Three projectiles A, B and C are thrown from the same point in the same plane. Their trajectories are shown in the figure. Then comment on time, launch speed and horizontal velocity of the particle.**

Ans. (A) The horizontal velocity of a projectile remains constant throughout the trajectory because no acceleration due to gravity acts on the horizontal component velocity of a projectile. Thus, no force to change its velocity. That's why the horizontal velocity remains constant in a projectile motion.

(B) At the highest point of trajectory of a projectile, the vertical component of the initial speed is zero, while the horizontal component of the initial speed remains unchanged. Thus at the highest point of the trajectory, the speed of the trajectory, the speed of the projectile is equal to the horizontal components of the initial speed.

(C) *(b)* 60°

Explanation: Plane is flying at a speed

$$= 600 \times \frac{5}{18}$$

$$= \frac{500}{3} \text{ m/s horizontally}$$

(at a height 1960 m)

Time taken by the kit to reach the ground,

$$t = \sqrt{\frac{2h}{g}}$$

$$= \sqrt{\frac{2 \times 1960}{9.8}} = 20 \text{ s}$$

In this time the kit will move horizontally by,

$$x = ut$$

$$= \frac{500}{3} \times 20 = \frac{10{,}000}{3} \text{ m}$$

So, the angle of sight

$$\tan\theta = \frac{x}{h}$$

$$= \frac{10{,}000}{3 \times 1960} = \frac{10}{5.88}$$

$$= 1.7 = \sqrt{3}$$

or $\theta = 60°$

(D) *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: For both the cases, the vertical component of the velocity is zero *i.e.* $u_y = 0$

Using $S_y = u_y t + \frac{1}{2} a_y t^2$

$$\therefore \quad h = 0 + \frac{ht^2}{2}$$

$$\Rightarrow \quad t = \sqrt{\frac{2h}{g}}$$

Hence the body will reach the ground at the same time.

Also from the equation used, the horizontal velocity has no effect on the vertical direction.

(E) The time of flight only depends on the vertical component of velocity and if A, B, C have the same maximum height that means they have the same vertical component of velocity *i.e.*, same time of flight. If C covers maximum horizontal distance at the same time as all three that means it has maximum horizontal velocity. If all of them have the same vertical velocity and C has maximum horizontal velocity that implies C has highest total velocity.

TOPIC 8

UNIFORM CIRCULAR MOTION

When a point object is moving on a circular path with a constant speed, *i.e.*, it covers equal distances on the circumference of the circle in equal intervals of time, then the motion of the object is said to be a uniform circular motion.

Let us consider a point object moving along a circular path. For uniform circular motion, when the object completes one revolution, the angle traced at its axis of circular motion is 2π radians, which implies,

when $t = T$, $\theta = 2$ radians.

$\therefore$ Angular velocity

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi n \text{ (since } T = \frac{1}{n}\text{)}$$

Where, n = frequency of a point object, T = time period.

Relation between linear velocity and angular velocity: $\omega = \frac{v}{r}$

where r is the radius of circular path.

ω = angular velocity

v = linear velocity

Angular acceleration of an object in circular motion is defined as the rate of change of its angular velocity.

If $\Delta\omega$ be the change in angular velocity of the object between the time interval Δt, the angular acceleration,

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

The S.I. unit of angular acceleration is rad. s^{-2}.

Relation between linear acceleration and angular acceleration:

$$\vec{a} = \vec{\alpha} \times \vec{r}$$

The acceleration of an object moving with speed v in a circle of radius r has a magnitude $\frac{v^2}{r}$ and is always directed towards the centre. This acceleration is called centripetal acceleration a_c.

Centripetal acceleration, $a_c = \frac{v^2}{r}$

$$= \omega^2 r = r(2\pi\nu)^2$$

$$= 4\pi^2 r\nu^2 = \frac{4\pi^2 r}{T^2}$$

Example 1.14: **An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity.**

Ans. Radius of the loop, r = 1 km = 1000 m

Speed of the aircraft, v = 900 km/h

$$= 900 \times \frac{5}{18}$$

$$= 250 \text{ m/s}$$

Centripetal acceleration, $a_c = \frac{v^2}{r}$

$$= \frac{250^2}{1000}$$

$$= 62.5 \text{ ms}^{-2}$$

Acceleration due to gravity, $g = 9.8$ m/s^2

$$a_c = \frac{62.5}{9.8} g$$

$$a_c = 6.38 \text{ g}$$

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. **The resultant of two vectors $\vec{A}$ and $\vec{B}$ is perpendicular to the vector $(\vec{A})$ and its magnitude is equal to half of the magnitude of the vector $\vec{B}$. The angle between $\vec{A}$ and $\vec{B}$ is:**

(a) 120° **(b)** 150°

(c) 135° **(d)** None of these

Ans. *(b) 150°*

Explanation:

$$\text{Cos } \beta = \frac{R}{B} = \frac{1}{2}$$

$\therefore$ $|A| = \frac{1}{2}|\beta|$

This means, $\beta = 60°$

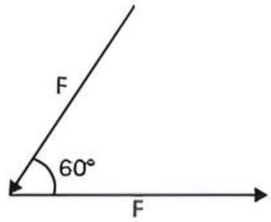
The angle between $\vec{A}$ and $\vec{B}$ = 90° + β

= 90° + 60° = 150°

Caution

➥ *Students should know that when you write R = A + B, then it is true that R is greater than A and B but while writing $\vec{R} = \vec{A} + \vec{B}$, the meaning is completely different i.e., $\vec{R}$ is not always greater than $\vec{A} + \vec{B}$.*

2. **Two forces, each equal to F act on the shown figure. Their resultant is:**



(a) $\frac{F}{2}$ **(b)** F

(c) $\sqrt{3}$ F **(d)** $\sqrt{5}$ F

Ans. *(b) F*

Explanation: The angle between the force,

$\theta = 120°$

and $\alpha = 180° - \theta = 60°$

So, resultant is, $R = \sqrt{A^2 + B^2 - 2AB\cos\alpha}$

$$= \sqrt{(F^2 + F^2 - 2F^2 \cos 60)}$$

$$= \sqrt{2F^2 - 2F^2 \times \frac{1}{2}}$$

$$= \sqrt{F^2} = F$$

Related Theory

➥ *During an equilibrium, there is no linear motion of the object. Therefore, the resultant force is zero. There is minimum potential energy of the object for stable equilibrium.*

3. A unit vector along the incident beam of light is $\hat{i}$. The unit vector for the related refracted beam of light is $\hat{r}$. $\hat{n}$ is a unit vector normal to the boundary of the medium. If μ be the refracted index of the medium, then Snell's Law of Refraction is:

(a) $\hat{i} \times \hat{n} = \mu(\hat{n} + \hat{r})$

(b) $\hat{i}.\hat{n} = \mu(\hat{r}.\hat{n})$

(c) $\hat{i} \times \hat{n} = \mu(\hat{r} \times \hat{n})$

(d) $\mu(\hat{i} \times \hat{n}) = \hat{r} \times \hat{n}$

Ans. *(c)* $\hat{i} \times \hat{n} = \mu(\hat{r} \times \hat{n})$

Explanation: As we know that, Snell's law is

$$\frac{\sin i}{\sin r} = \mu$$

$$|\hat{i} \times \hat{n}| = |\hat{i}||\hat{n}| \sin i$$

$$|\hat{r} \times \hat{n}| = |\hat{r}||\hat{n}| \sin r$$

$$\sin i = \mu \sin r$$

$$|\hat{i} \times \hat{n}| = \mu |\hat{r} \times \hat{n}|$$

Since

$$\hat{i} \times \hat{n} || \hat{r} \times \hat{n}$$

$$\hat{i} \times \hat{n} = \mu(\hat{r} \times \hat{n})$$

Related Theory

➥ *Unit vector has no unit and dimensions. It is used to specify a direction only.*

4. A body moves 6 m North, 8 m east and 10 m vertically upwards, the resultant displacement from its initial position is:

(a) $10\sqrt{2}$ m (b) 10 m

(c) $\frac{10}{\sqrt{2}}$ m (d) 20 m

Ans. *(a)* $10\sqrt{2}$ *m*

Explanation: According to the question,

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$= \sqrt{6^2 + 8^2 + 10^2}$$

$$= 10\sqrt{2} \text{ m}$$

Caution

➥ *Students should always label the displacement vectors and denote the result of their sum as resultant vector. If the value of direction is in negative, the person has walked in negative direction. The value is not always supposed to be subtracted in such cases.*

5. A vector is not changed if:
(a) it is rotated through an arbitrary angle
(b) it is multiplied by an arbitrary scalar
(c) it is cross multiplied by a unit vector
(d) it is displaced parallel to itself.

Ans. *(d) it is displaced parallel to itself.*

Explanation: A vector has both magnitude and direction. A vector is changed if it is through certain angles as its direction has been changed, if a vector is multiplied by scalar quantity its magnitude is changed, a vector does not change if it is displaced parallel to itself.

Caution

➥ *Students must know that only similar quantities can be added or subtracted. Quantities with same unit can be added and it doesn't not make any sense when we talk of adding mass of body to temperature of the body.*

6. Vector $\vec{A}$ is 2 cm long and is 60° above the x-axis in the first quadrant. Vector $\vec{B}$ is 2 cm long and is 60° below the x-axis in the fourth quadrant. The sum $\vec{A} + \vec{B}$ is a vector of magnitude:
(a) 2 cm along +y-axis
(b) 2 cm along +x-axis
(c) 2 cm along –y-axis
(d) 2 cm along –x-axis

Ans. *(b) 2 cm along +x-axis.*

Explanation: From given data,

$$\vec{A} = 2\cos 60\hat{i} + 2\sin 60\hat{j}$$

$$\vec{B} = 2\cos(-60)\hat{i} + 2\sin(-60)\hat{j}$$

$$= 2\cos 60\hat{i} - 2\sin 60\hat{j}$$

$$\vec{A} + \vec{B} = 2 \times 2\cos 60\hat{i} + 0\hat{j}$$

$$= 4.\frac{1}{2}\hat{i} = 2\hat{i}$$

$$|\vec{A} + \vec{B}| = 2$$

7. ABCDEF is a regular hexagon with point O as centre. The value of $\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF}$ is:

(a) $2\vec{AO}$ (b) $4\vec{AO}$

(c) $6\vec{AO}$ (d) 0

Ans. *(c)* $6\vec{AO}$

Explanation: In Hexagon;

$$\therefore \quad \vec{AB} + \vec{AF} = \vec{AO}$$

$$\vec{AB} = \vec{AO} - \vec{AF}$$

$$\vec{AC} = \vec{AB} - \vec{AO}$$

$\therefore \quad \vec{AD} = 2\vec{AO}$

$\vec{AE} = \vec{AO} + \vec{AF}$

Now,

$\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF} = 5\vec{AO} + \vec{AB} + \vec{AF}$

$= 5\vec{AO} + \vec{AO} = 6\vec{AO}$

8. In going from one city to another, a car travels 75 km North, 60 km North-west and 20 km east. The magnitude of displacement between two cities is (take $\sqrt{2} = 0.7$):

(a) 170 km **(b) 137 km**

(c) 119 km **(d) 140 km**

Ans. *(c) 119 km*

Explanation: Displacement of car

$= 75\hat{j}$ (75 km North) $- 60 \cos 45\, \hat{i} + 60 \sin 45\, \hat{j}$ (60 km North – west) + 20 (20 km east)

Final Location

$$= \left(\frac{-60}{\sqrt{2}} + 20\right)\hat{i} + \left(\frac{60}{\sqrt{2}} + 75\right)\hat{j}$$

$$= (-60 \times 0.7 + 20)\hat{i} + (42 + 75)\hat{j}$$

$$= -22\hat{i} + 117\hat{j}$$

So, displacement is

$$= \sqrt{(-22)^2 + (117)^2}$$

$$= \sqrt{14173} = 119 \text{ km}$$

Caution

- *Students should remember that dimension of new vectors quantity = Dimension of scalar quantity × Dimensions of vector quantity.*
- *In above case, if the final value is negative, it would not mean that displacement is negative as it would be impossible. In such cases always take modulus of the final answer and write the answer in positive.*

9. On a horizontal flat ground, a person is standing at point A. At this point he installs a 5 m long pole vertically. Now he moves 5m towards east and then 2 m towards pole to the top of the second pole. Find the displacement and magnitude of the bird.

(a) $5\hat{i} + 2\hat{j} - 2\hat{k}$, $\sqrt{33}$ m

(b) $6\hat{i} + 2\hat{j} - 2\hat{k}$, $\sqrt{23}$ m

(c) $5\hat{i} + 3\hat{j} - 2\hat{k}$, $\sqrt{23}$ m

(d) $5\hat{i} + 2\hat{j} - 2\hat{k}$, $\sqrt{23}$ m

Ans. *(a) $5\hat{i} + 2\hat{j} - 2\hat{k}$, $\sqrt{33}$ m*

Explanation:

A man install pole

AP = 5 m

another pole of 3 m be install at point B,

BQ = 3 m

$\vec{AP} = 5\hat{k}$

$\vec{AQ} = 5\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{AQ} = \vec{AQ} - \vec{AP}$

$= 5\hat{i} + 2\hat{j} + 3\hat{k} - 5\hat{k}$

displacement vector: $\vec{PQ} = 5\hat{i} + 2\hat{j} + 2\hat{k}$

For magnitude $\quad |\vec{PQ}| = \sqrt{5^2 + 2^2 + 2^2}$

$|\vec{PQ}| = \sqrt{33}$ m

10. An athlete throws a javelin with velocity of 20 ms^{-1} at an angle of 15° with the horizontal. How far the javelin would hit the ground from the point of projection:

(a) 20 m **(b) 20.2 m**

(c) 20.4 m **(d) 20.6 m**

Ans. *(c) 20.4 m*

Explanation: Range of javelin,

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{(20)^2 \sin 2 \times 15}{9.8}$$

$$= \frac{400 \times \sin 30°}{9.8}$$

$$= 20.4 \text{ m}$$

Related Theory

- *Horizontal and vertical components in projectile motion were stated by Galilea. He is also responsible for discovery of time. In projectile motion, time of ascent is equal to the time of descent.*

11. At the upper most part of a projectile, velocity and acceleration are at an angle of:

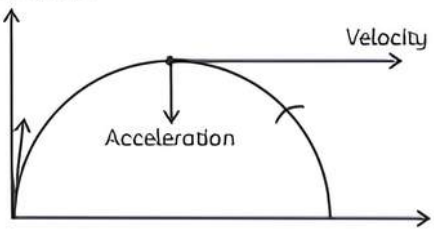
(a) 0 degree **(b) 30 degree**

(c) 180 degree **(d) 90 degree**

[Delhi Gov. SQP 2022]

Ans. *(d) 90 degree*

Explanation:



As seen in the diagram, velocity impacts horizontally, whereas gravity's acceleration acts vertically. As a result, the angle between them is 90°.

12. A soldier fires a bullet horizontally from the top of a cliff with a velocity of 10 ms^{-1}. If the bullet strikes the ground after 2 s. Find the height of the cliff, also calculate the velocity with which the bullet strikes the ground.

(a) 19.6 m, 22.01 ms^{-1}
(b) 22.01 m, 19.6 ms^{-1}
(c) 19 m, 22 ms^{-1}
(d) None of the above

Ans. *(a) 19.6 m, 22.01 ms^{-1}*

Explanation: The height of the cliff, using

$$y = v_0 t + \frac{1}{2} a_y t^2$$

$$y = 0 + \frac{1}{2} \times 9.8 \times (4)^2 = 19.6 \text{ m}$$

So, the height of cliff is 19.6 m.

And for velocity, we know that.

$$v = \sqrt{u^2 + 2gh}$$

$$= \sqrt{(10)^2 + 2 \times 9.8 \times 19.6}$$

$$= \sqrt{484.16}$$

$$= 22.01 \text{ ms}^{-1}$$

13. A person aims a gun at a bird from a point at a horizontal distance of 100 m. If the gun can impart a speed of 500 ms^{-1} to the bullet, at what height above the bird he must aim his gun in order to hit it?

(a) 10 cm (b) 20 cm
(c) 30 cm (d) 50 cm

Ans. *(b) 20 cm*

Explanation: Time taken by bullet to cover

$$\text{horizontal distance} = \frac{100}{500} = 0.2 \text{ sec.}$$

Distance covered by bullet in same time

$= 0.5 \times 10 \times 0.2 \times 02 = 0.2 \text{ m} = 20 \text{ cm}$

Person should aim 20 cm above to hit the bird.

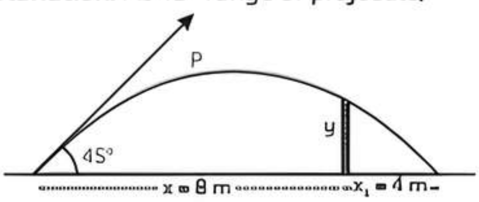
14. A particle is projected at an angle of 45°, 8 m away from the foot of a wall, just touches the top of the wall and falls on the ground on the opposite side at a distance 4 m from it. The height of wall is:

(a) $\frac{2}{3}$ m (b) $\frac{4}{3}$ m
(c) $\frac{8}{3}$ m (d) $\frac{3}{4}$ m

[NCERT Exemplar]

Ans. *(c) $\frac{8}{3}$ m*

Explanation: At 45° range of projectile.



$$R = 8 + 4 = 12 \text{ m}$$

$$\text{Height of wall, } y = x\tan\theta \;\; \frac{gx^2}{2u^2\cos^2\theta}$$

$$y = (x\tan\theta)\left[1 - \frac{x}{R}\right]$$

$$= 8 \times 1 \times \left[1 - \frac{8}{12}\right]$$

$$= \frac{8}{3} \text{ m}$$

15. An arrow is shot into the air. Its range is 200 m and its time of flight is 5 s. If g = 10 ms^{-2}, then the horizontal component of velocity of the arrow is

(a) 25 m/s (b) 40 m/s
(c) 31.25 m/s (d) 12.5 m/s

Ans. *(b) 40 m/s*

Explanation: The time taken by the projectile to reach the ground is calculated above. During this time the projectile moves in the horizontal direction and covers a certain distance. This distance is called range. The horizontal range covered is given by the equation,

$$R = u_x T$$

$$u_x = \frac{R}{T}$$

$$= \frac{200}{5} = 40 \text{ m/s}$$

Related Theory

In real life, air causes resistance on projectile motion. Not only air, but force due to viscosity and friction also act as dissipative forces.

16. Two vectors, both equal in magnitude have their resultant equal in magnitude of either. The angle between these vectors will be :

(a) 90 degree (b) 0 degree
(c) 120 degree (d) 180 degree

[Delhi Gov. SQP 2022]

Ans. *(c) 120 degree*

Explanation: This is an important result to remember. Two vectors at 120 degree produce a vector equal to their magnitude. Let their magnitude be a.

$$\text{Resultant} = a = \sqrt{(a^2 + a^2 + 2 \times a \times a \times \cos\theta)}$$

$$= a\sqrt{(2 + 2\cos\theta)}$$

$$\Rightarrow \quad \cos\theta = -\frac{1}{2}$$

$$\Rightarrow \quad \theta = 120°$$

17. A stuntman plans to run along a roof top and then horizontally off it to land on the roof of the next building. The roof of the next building is 4.9 m below the first one and 6.2 m away from it. What should be his minimum roof top speed in m/s, so that he can successfully make the jump?

(a) 3.1 (b) 4.0

(c) 4.9 (d) 6.2

Ans. *(d) 6.2*

Explanation: Time taken to cover the depth 4.9.

$$m = \sqrt{\frac{2h}{g}} \times u$$

$$u = m \times \sqrt{\frac{g}{2h}}$$

$$= 6.2 \times \sqrt{\frac{9.8}{2 \times 4.9}}$$

$$= 6.2 \text{ m/s}$$

18. A boy wants to jump from building A to building B. Height of building A is 25 m and that of building B is 5 m. Distance between buildings is 4 m. Assume that the boy jumps horizontally then minimum velocity with which he has to jump to land safely on building B is:

(a) 6 m/s (b) 8 m/s

(c) 4 m/s (d) 2 m/s

Ans. *(d) 2 m/s*

Explanation: $t = \sqrt{\frac{2 \times 20}{g}} = 2 \text{ s}$

$$x = v_x t$$

$$\Rightarrow \quad v_x = \frac{4}{2}$$

$$= 2 \text{ m/s}$$

19. Preeti reached the metro station and found that the escalator was not working. She walked up the stationary escalator in time t_1. On the other days if she remains stationary on the moving escalator, then the escalator takes her up in time t_2. The time taken by her to walk up on the moving escalator will be:

(a) $\frac{t_1 t_2}{t_2 - t_1}$ (b) $\frac{t_1 t_2}{t_2 + t_1}$

(c) $t_1 - t_2$ (d) $\frac{t_1 + t_2}{2}$

Ans. *(b)* $\frac{t_1 t_2}{t_2 + t_1}$

Explanation: Let, v_1 = velocity of Preeti.

v_2 = velocity of escalator.

l = distance

$$t = \frac{l}{v_1 + v_2} = \frac{l}{\frac{l}{t_1} + \frac{l}{t_2}}$$

$$= \frac{t_1 t_2}{t_1 + t_2}$$

20. At what angle, do the forces (P+Q) and (P–Q) act so that the resultant is $\sqrt{3P^2 + Q^2}$?

(a) 60° (b) 120°

(c) 180° (d) 0°

Ans. *(a) 60°*

Explanation: A = P + Q B = P – Q and

$R = \sqrt{3P^2} + Q^2$

And $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

So. $\sqrt{3P^2 + Q^2)^2} = (P+Q)^2 + (P-Q)^2$

$+ 2(P+Q)(P-Q)\cos\theta$

$$\cos\theta = \frac{P^2 - Q^2}{2(P^2 - Q^2)} = \frac{1}{2}$$

$$Q = \cos^{-1}\left(\frac{1}{2}\right) = 60°$$

Assertion-Reason Questions

Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these question from the codes (a), (b), (c) and (d) as given below:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

21. Assertion (A): Minimum number of non-equal vectors in a plane to give zero resultant is three.

Reason (R): If $\vec{A} + \vec{B} + \vec{C} = \vec{O}$, then they must lie in one plane.

[Delhi Gov. QB 2022]

Ans. *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation: For a zero resultant, the provided vectors must be arranged along the sides of a closed polygon, and the least number of sides of a polygon is three.

22. Assertion (A): A physical quantity cannot be called as a vector if it has zero magnitude.

Reason (R): A scalar has both magnitude and direction.

[Delhi Gov. SQP 2022]

Ans. *(d) A is false and R is also false.*

Explanation: A physical quantity with no magnitude is known as a vector, and more specifically, a null vector.

A scalar quantity just has magnitude and no direction. For instance, speed, time, and so on.

23. **Assertion (A):** The magnitude of the sum of two vectors is always greater than the magnitude of their subtraction.

Reason (R): At θ = 90°, addition and subtraction of vectors are unequal.

Ans. *(d) A is false and R is also false.*

Explanation:

The addition of two vectors is,

$$|A+B| = \sqrt{A^2+B^2+2AB\cos\theta}$$

Its subtraction is,

$$|A-B| = \sqrt{A^2+B^2-2AB\cos\theta}$$

So, when then $90° < \theta < 270°$,

Then, $|A + B| < |A - B|$

Thus, the vector addition of two vectors is not always greater than their vector subtraction.

Also $\theta = 90°$

$$|A + B| = |A - B| = \sqrt{A^2+B^2}$$

24. **Assertion (A):** When a particle moves with uniform velocity, its displacement may increase or decrease.

Reason (R): In successive time intervals, if the average velocities of a particle are equal then the particle are equal then the particle must be moving with constant velocity.

Ans. *(c) A is true but R is false.*

Explanation: For a moving body, velocity is non-zero. If velocity is constant, displacement will keep increasing and hence, average velocity cannot be zero for a body under uniform motion. For average velocity to be zero, the body should either be at rest which is false as per the statement of the reason or it should be able to return to its initial position which requires a change in velocity.

Average velocity is given by the ratio of change in displacement to change in time.

When a particle is thrown upwards, it is under a uniform acceleration equal to the acceleration of gravity. After some time, it returns to its point of projection. In this time interval, net displacement is zero and hence average velocity is zero.

25. **Assertion (A):** Horizontal range is same for angle of projection θ and (90 – θ).

Reason (R): Horizontal range is independent of angle of projection.

[Delhi Gov. SQP 2022]

Ans. *(c) A is true and R is false.*

Explanation: Range of projectile is given by:

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$R_1 = \frac{u^2 \sin 2\theta}{g}$$

$$R_2 = \frac{u^2 \sin 2(90° - \theta)}{g}$$

$$= \frac{u^2 \sin 2\theta}{g}$$

$$\Rightarrow \quad R_1 = R_2$$

Horizontal range depends upon angle of projection and it is same for complementary angles *i.e.*, θ and (90° – θ).

26. **Assertion (A):** A man cross river of width *d* in minimum time. On increasing the river velocity, the minimum time to cross the river by man will remain unchanged.

Reason (R): As velocity of a river is perpendicular to its width, so time to cross the river by man will remain unchanged.

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: If the direction of river flow and direction of swimming is perpendicular to each other, the component speed of the swimmer will be maximum and it would be V sin 90°.

27. **Assertion (A):** Two balls of different masses are thrown vertically upward with the same speed. They will pass through their point of projection in the downward direction with the same speed.

Reason (R): The maximum height and downward velocity attained at the point of projection are independent of the mass of the ball.

[Delhi Gov. SQP 2022]

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: Let the balls be thrown vertically upward with speed *u*.

Using $\quad S = ut + \frac{1}{2}at^2,$

where $\quad a = -g$

$\therefore$ The maximum height, $H = ut - \frac{gt^2}{2}$

Also using $v^2 - u^2 = 2aS$

$\therefore \quad v^2 - u^2 = 2(-g)(0)$

$\Rightarrow \quad v = u$

Thus, the balls will pass through the point of projection in downward direction with same speed (v).

Also, from the equation used, the maximum height and the downward velocity attained are independent of mass of the ball.

28. Assertion (A): If a body is dropped from the top of a tower of a height h and another body is thrown up simultaneously with uniform velocity v from the foot of the tower then both of them would meet after a time $\frac{h}{u}$.

Reason (R): For a body projected upwards, the distance covered by the body in the last second of its upward journey is always 4.9 m irrespective of velocity of projection

Ans. *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation: A body thrown upwards will decelerate downwards at 1 g = 9.8 m/s²

When it reaches the apex of the throw, it will be at rest. During the last second, it decelerated at 1 g for 1 second.

To calculate how far it travelled in the 1 second at 1 g deceleration, we can calculate how far it would travel from rest in 1 s at 9.8 m/s² using the formula:

$$s = ut + \frac{1}{2}at^2$$

s = distance travelled, u = start speed (0), a = acceleration = 9.8 m/s², t = time = 1 sec which gives value of,

$$= 0 \times 1 + \frac{1}{2} \times 9.8 \times 1^2$$

$$= 4.9 \text{ m}$$

29. Assertion (A): The magnitude of the resultant of two vectors may be less than the magnitude of either vector.

Reason (R): Vector addition is commutative.

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: The resultant of two vectors is:

If θ is an obtuse angle, then the magnitude of R will be less than the magnitude of either vector A or B. If vectors are in opposite directions and are equal in magnitude, then also the magnitude of R will be less than the magnitude of either vectors A or B. And vector addition is always commutative in nature.

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

30. Assertion (A): In projectile motion, when the horizontal range is n times the maximum height, the angle of projection is given by $\tan\theta = \frac{4}{n}$.

Reason (R): In the case of horizontal projection the magnitude of vertical velocity increases with time.

Ans. *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation:

$$\vec{u} = u\hat{i} + u\hat{j}$$

$$\vec{v}t = u\hat{i} + (o - g(t))\hat{j}$$

$$= u\hat{i} - g(t)\hat{j}$$

$$\text{Range} = \frac{2v^2\sin\theta\cos\theta}{g}$$

$$\text{Maximum height} = \frac{v^2\sin^2\theta}{2g}$$

$$\frac{2v^2\sin\theta\cos\theta}{g} = \frac{n(v^2\sin^2\theta)}{2g}$$

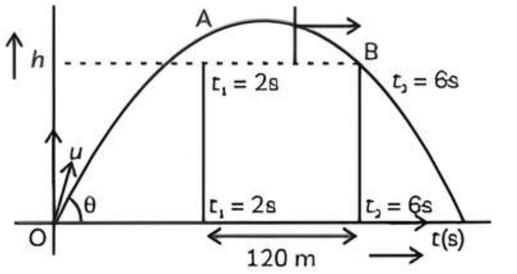
$$\tan\theta = \frac{4}{n}$$

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

31. *A projectile is projected from a point O on the ground with an initial velocity u at an elevation angle θ from the horizontal direction as shown in the figure. It just crosses two walls A and B of same height h situated symmetrically at times t_1 = 2s and t_2 = 6 s respectively. The horizontal distance between the two walls is d = 120 m. (take g = 10 m/s²)*



[Delhi Gov. QB 2022]

(A) The projectile motion is an example of:

(a) one dimensional motion

(b) two dimensional motion

(c) three dimensional motion

(d) cannot say, precisely.

(B) The total time of flight of the projectile:

(a) 8 s (b) 10 s

(c) 4 s (d) 12 s

(C) The value of angle of projection θ of the projectile is:

(a) $\tan^{-1}\left(\frac{3}{4}\right)$ (b) $\tan^{-1}\left(\frac{4}{5}\right)$

(c) $\tan^{-1}\left(\frac{4}{3}\right)$ (d) $\tan^{-1}\left(\frac{5}{4}\right)$

(D) The projectile velocity *u* of the projectile is:

(a) 30 m/s (b) 40 m/s

(c) 50 m/s (d) $20\sqrt{3}$ m/s

(E) The height *h* of either of two walls is:

(a) 120 m (b) 30 m

(c) 15 m (d) 60 m

Ans. (A) *(b) two dimensional motion*

(B) *(a) 8 s*

Explanation: If a projectile passes a certain height *h* at two times t_1 and t_2 then, $t_1 + t_2$ = total time of flight = T

$$T = t_1 + t_2$$
$$= 2 + 6 = 8 \text{ sec.}$$

(C) *(c)* $\tan^{-1}\left(\frac{4}{3}\right)$

Explanation: We know that

$$T = \frac{2u\sin\theta}{g}$$

$$\Rightarrow \quad u\sin\theta = \frac{gT}{2}$$

$$= \frac{10\times 8}{2} = 40$$

horizontal distance between the walls *d*

Then, $d = (u\cos\theta)(t_1 - t_2)$

$$u\cos\theta = \frac{d}{t_2 - t_1}$$

$$= \frac{120}{6-2} = 30$$

Now, $\frac{u\sin\theta}{u\cos\theta} = \frac{40}{30}$

$$\Rightarrow \quad \tan\theta = \frac{4}{3}$$

$$\therefore \quad \theta = \tan^{-1}\frac{4}{3}$$

(D) *(c) 50 m/s*

Explanation:

$$\tan\theta = \frac{4}{3}, \ \sin\theta = \frac{4}{\sqrt{4^2+3^2}} = \frac{4}{5}$$

$$u\sin\theta = 40$$

$$u \times \frac{4}{5} = 40 \Rightarrow 50 \text{ m/s}$$

(E) *(d) 60 m*

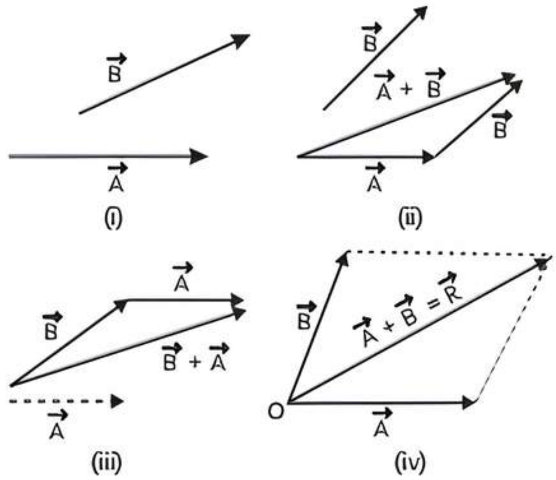
Explanation:

$$h = (u\sin\theta)t_1 - \frac{1}{2}gt_1^2$$

$$h = 40 \times 2 - \frac{1}{2} \times 10(2)^2$$

$$= 60 \text{ m}$$

32. *Parallelogram law of vector addition- states that if two vectors are represented in magnitude and direction by two adjacent sides of a parallelogram drawn from a point then their resultant is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point.*



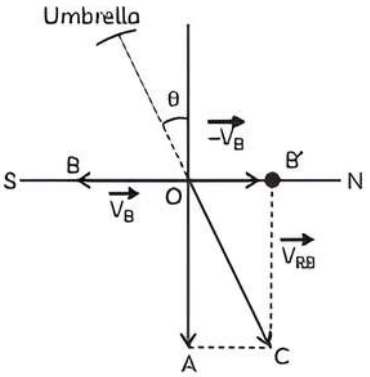
(A) State whether the statements are true or false.

(i) If the angle between two-unit vectors is 120° then, their resultant is another unit vector.

(ii) A scalar quantity is one that has values for observers with different orientation of the axis.

(B) Four forces are acting on a point. The first force is 200 N acting due north, the second is 100 N acting due south, the third force is 500 N acting due east and the fourth force is 300 N acting due west. What is the magnitude and direction of the resultant force?

(C) Rain in falling vertically with a speed of 30 ms^{-1}. A woman on a bicycle is travelling with a speed of in the north to south direction. In what direction should she hold an umbrella in order to protect herself from rain?



Ans. (A) (i) True

Explanation: let $|\vec{A}| = x$ and $|\vec{B}| = x$

$\therefore |\vec{R}| = x$

Using, $R^2 = A^2 + B^2 + 2AB\cos\theta$

$x^2 = x^2 + x^2 + 2(x)(x)\cos\theta$

$\cos\theta = -\frac{1}{2}$

So, $\theta = +120°$

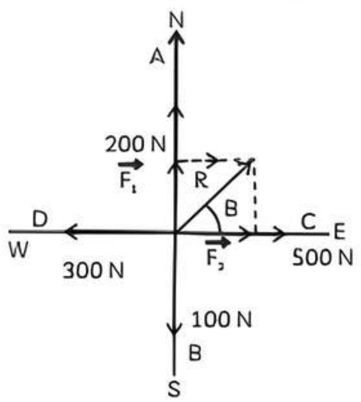
(ii) False.

Explanation: The value of scalar quantities (for example- body of a man) measured by observers with different orientation of axes remain unchanged.

(B) Resultant of $\vec{OA}$ and $\vec{OB}$.

$\vec{F_1} = 200 - 100$

$= 100$ N, along the north.



Resultant of $\vec{OC}$ and $\vec{OD}$.

$\vec{F_2} = 500 - 300$

$= 200$ N due east

Now $\vec{F_1}$ and $\vec{F_2}$ are at right angles to each other and its resultant $\vec{F}$ is,

$\vec{F} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$= \sqrt{(100)^2 + (200)^2 + 2(100)(200)\cos 90°}$

$\vec{F} = 253$ N

Angle made by F with F_2

$\tan\beta = \frac{F_1}{F_2} = \frac{100}{100} = 0.5$

$\beta = \tan^{-1}\left(\frac{1}{2}\right) = 26°34'$.

the direction is north of east.

(C) Velocity of rain $(\vec{v}_R) = 30$ ms^{-1}

Velocity of bicycle $(\vec{v}_B) = 10$ ms^{-1}

To protect herself, the direction of umbrella should be in direction to relative velocity

i.e. $\vec{v}_{RB} = \vec{v}_R + (-\vec{v}_B)$

Then angle that $\vec{v}_{RB}$ makes with the vertical

Then $\tan\theta = \frac{AC}{OA} = \frac{|-\vec{v}_B|}{|\vec{v}_R|} = \frac{10}{30} = \frac{1}{3}$

$\theta = \tan^{-1}\frac{1}{3} = 18°$

Then women hold her umbrellas in a vertical plane containing N-S direction at an angle of about 18° with the vertical towards south.

33. *During a take-off, a plane has to attain a specific speed before it can lift off the runway. Every plane has a different speed to achieve depending on their shape, size and total mass. On smaller airports, the runway is smaller therefore, the acceleration is the key. For theoretical purposes, we just consider the length of the runway, initial and final velocity and acceleration but in real world, there are other factors which affect the take-off of a plane like temperature, wind speed, etc.*

(A) Suppose forces of 12 N and 3 N are exerting on a body and its resultant is 13.74 N, then the angle between two forces to get the resultant of 13.74 N is:

(a) 30° (b) 60°
(c) 90° (d) 120°

(B) A car moving at the speed of 40 km/h can be stopped by applying brakes after at least 2 m. The same car is moving with the speed of 80 km/h. What is the minimum stopping distance?

(a) $\frac{20}{3}$ m (b) 20 m
(c) 60 m (d) 180 m

(C) The initial velocity of a particle is u (at $t = 0$) and the acceleration is given by $f = at$, which of the following relations are valid?

(a) $v = u + at^2$ (b) $v = u + \frac{at^2}{2}$
(c) $v = u + at$ (d) $v = u$

(D) The velocity of a particle moving with constant acceleration at an instant t_0 is 10 m/s. After 5 seconds of that instant, the velocity of the particle is 20 m/s. The velocity at 3 second before t_0 is:

(a) 8 m/s (b) 4 m/s
(c) 6 m/s (d) 7 m/s

(E) A helicopter dropped rations, medicines, and other goods in a flood-stricken region. At a height of 98 meters above the surface, the helicopter was flying. Authorities were assisted in evacuating the victims by students from a local school. They noticed a toddler who was drowning. They ran to the boy with the lifeboat and saved him. What is the time taken by the objects dropped from a helicopter to reach the ground?

(a) 4.5 s (b) 5.2 s
(c) 2.5 s (d) 5.0 s

Ans. (A) *(b) 60°*

Explanation: We know,

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Here, $R = 13.74$, $A = 12$ N, $B = 3$ N, $\theta = ?$

So, $(13.74)^2 = (12)^2 + (3)^2 + 2(12)(3)\cos\theta$

$(13.74)^2 - 153 = 72\cos\theta$

$35.70 = 72\cos\theta$

$35.70 = 72\cos\theta$

$$\cos\theta = \frac{1}{2}$$

Or $\theta = \cos^{-1}\left(\frac{1}{2}\right)$

$= 60°$

(B) *(d) 180 m*

Explanation: We know that, $s \propto u^2$,

$$\frac{s_1}{s_2} = \frac{u_1^2}{u_2^2}$$

$$\frac{20}{s_2} = \frac{10^2}{30^2}$$

$$s_2 = 180 \text{ m}$$

(C) *(b)* $v = u + \frac{at^2}{2}$

Explanation: As given $f = at = \frac{dv}{dt}$

$$\int_u^v dv = \int_0^t at\,dt$$

$$v - u = \frac{at^2}{2}$$

$$v = u + \frac{at^2}{2}$$

(D) *(b) 4 m/s*

Explanation: The acceleration of the particle is,

$$a = \frac{20-10}{5} = 2 \text{ m/s}^2$$

For 3 seconds before t_0,

$v = 10$ ms^{-1}; $t = 3$ s; $a = 2$ ms^{-2} $u = ?$

$$v = u + at$$

$$10 = u + 2(3)$$

Or $u = 10 - (2)(3) = 4$ m/s

(E) *(a) 4.5 s*

Explanation: The time taken by the objects dropped from helicopter to reach the ground is:

$$h = v_i t + \frac{1}{2}gt^2$$

If $v_i = 0$,

then $t = \frac{\sqrt{2h}}{g}$

$$t = \sqrt{\frac{2 \times 98}{9.8}}$$

$$= \sqrt{20} = 4.5 \text{ s}$$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

34. A gunman always keep his gun slightly tilted above the line of sight while shooting. Why?

[Delhi Gov. QB 2022]

Ans. When a bullet is shot from a gun with the barrel aimed at the target, it begins to descend downward owing to gravity's acceleration. As a result, the bullet misses the target. To avoid this, the gun's barrel is aligned slightly above the target, so that the bullet, after travelling in a parabolic route, strikes the far target.

35. (A) What is the angle between A × B and B × A?

(B) Can the resultant of two vectors of different magnitudes be zero? Justify.

[Delhi Gov. SQP 2022]

Ans. (A) (A × B) = – (B × A) which are equal and opposite in direction.

Hence, it will have angle in between 180°.

(B) Let two vector $\vec{A}$ and $\vec{B}$ of different magnitude.

Their resultant is given by

$$\vec{R} = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta}$$

as $$|\vec{A}| \neq |\vec{B}|$$

Thus, $|\vec{A}|^2 + |\vec{B}|^2 \neq -2|\vec{A}| + |\vec{B}|\cos\theta$

Thus, the resultant of two vectors of different magnitude cannot be zero.

Three vectors are added to get resultant zero.

The essential condition for resultant of three vector to be zero is that they must be coplanar.

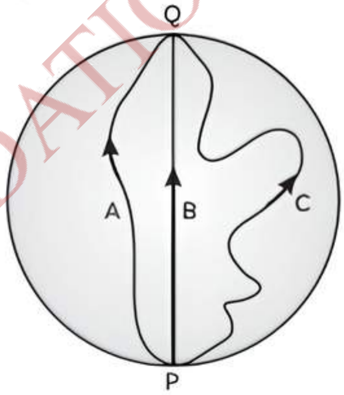
36. A body moves in a straight path for a period of time before returning to its original position. What is the average velocity of the body if the total time it takes to complete this course is 20 seconds and the total distance travelled is 40 metres?

Ans. When the body returns to its starting point then its displacement is zero, so average velocity is also zero.

37. When a vector is multiplied by a real positive number, a new vector is obtained and its magnitude and direction are also obtained. Deduce what is the value of obtained magnitude and direction of the new vector.

Ans. The magnitude of the new vector which is the result of multiplication by a real positive number is equal to the product of the real positive number and magnitude of the original vector. However, the direction is the same as that of the direction of the original vector.

38. Three girls are skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Fig. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of the path skate?



[Delhi Gov. SQP 2022]

Ans. The shortest distance between a particle's starting and final locations defines displacement. In the aforementioned case in question, all of the girls begin at point P and work their way to point Q. Their displacement magnitudes will be equal to the diameter of the ground.

The radius of the ground = 200 m

Diameter of the ground = 2 × 200

= 400 m

As a result, the magnitude of displacement for each girl is 400 m. This is the exact length of the path skated by girl B.

39. Suppose two vectors are placed in such a way that they are parallel to each other, then the effect on the values of magnitude and direction of both the vectors?

Ans. The value of the magnitude of both the vectors which are parallel to each other is always equal and both are in same direction.

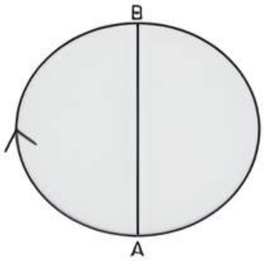
40. Does a vector have a location in space in addition to the magnitude and direction?

Can two vectors $\vec{P}$ and $\vec{Q}$ at a different location in space have an identical physical effect?

Ans. Yes, each vector has a location in space in addition to magnitude and direction. Two equal vectors having different locations may not have the same physical effect.

41. An athlete runs along the circular path of radius *r*. He starts running from a point and reaches just opposite to his starting point on the circular path. What is the magnitude of displacement of the athlete? **[NCERT Exemplar]**

Ans. Let the path covered by an athlete is from A to B.



The distance travelled $= \frac{1}{2} \times$ circumference of circular path

$$= \frac{1}{2} \times 2\pi r = \pi r$$

Magnitude of displacement $= |\vec{AB}| = 2r$

42. Can two vectors quantities be added in such a way that their resultant is zero? **[Diksha]**

Ans. Only under one condition, the two vector quantities can be combined to give zero resultant, only if they have equal magnitude and direction, are opposite to each other.

43. What is the maximum number of component into which a vector can be resolved? **[Delhi Gov. QB 2022]**

Ans. This vector is made up of any number of vectors whose outcome is the original vector. As a result, we have an endless number of options.

44. When a vector is multiplied by a scalar physical quantity, then its nature changes. Prove the statement with an appropriate example. **[Diksha]**

Ans. When an acceleration (vector) is multiplied by a body (scalar) of a man it gives a force (vector) quantity whose nature is different from acceleration, this proves that when a vector quantity is multiplied by a scalar quantity its nature changes.

45. Can the magnitude of the resultant vector of two vectors be less than the magnitude of any of the given vectors?

Ans. Yes, if the angle between the vector is more than 90°.

46. There are two displacement vectors, one of magnitude 3 metres and the other 4 metres. How should the two vectors be added so that the magnitude of a resultant vector be 7 meters, 1 meter and 5 meters.

Ans. For 7 metres, $\theta = 0°$ *i.e.*, is in same direction.

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{3^2 + 4^2 + 2(3)(4)\cos 0°}$$

$$= 7 \text{ m}$$

For 1 metre, 180° *i.e.*, is in opposite direction.

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{3^2 + 4^2 + 2(3)(4)\cos 180°}$$

$$= -1 \text{ m}$$

For 5 metre, $\theta = 90°$

$$R = \sqrt{3^2 + 4^2 + 2(3)(4)\cos 90°}$$

$$= 5 \text{ m}$$

47. What is the angle between velocity vector and acceleration vector in uniform circular motion? **[Delhi Gov. QB 2022]**

Ans. Because it is induced by centripetal force, the acceleration of the body in uniform circular motion is known as centripetal acceleration. Because the centripetal force is constantly directed towards the circular path's centre, the centripetal acceleration is likewise directed along it. In circular motion, the body's velocity is always along the tangent to the circular route. As a result, the angle formed by velocity and acceleration for an object travelling in uniform circular motion is 90 degrees.

48. The *x* and *y* coordinates of the particle at any time t is given by $x = 8t + 2t^2$ and $y = 15t$ where, *x* and *y* are in meters and *t* in seconds. What is the acceleration of the particle at 3 s? **[NCERT Exemplar]**

Ans. We know that the total acceleration of the particle is,

$$a = \sqrt{a_x^2 + a_y^2}$$

So, $$v_x = \frac{d(8t + 2t^2)}{dt} = 8 + 4t \text{ and}$$

$$a_x = \frac{d(8 + 4t)}{dt} = 4 \text{ ms}^{-1}$$

And $$a_y = \frac{d(15t)}{dt} = 0 \text{ ms}^{-1}$$

So, the acceleration of the particle at 3 s,

$$a = \sqrt{4^2 + 0^2} = 4 \text{ ms}^{-1}$$

49. What is the unit vector perpendicular to the plane of vectors $\vec{A}$ and $\vec{B}$? If $\vec{A} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = 2\hat{i} + \hat{k}$, find a unit vector perpendicular to plane of $\vec{A}$ and $\vec{B}$. **[Delhi Gov. QB 2022]**

Ans. A unit vector perpendicular to plane:

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}, \quad \hat{n} = \frac{2\hat{i} - 3\hat{j} - 4\hat{k}}{\sqrt{29}}$$

50. A stone dropped from a fixed rail line carriage's window takes 2 seconds to reach the ground. When the carriage is going at a constant pace, when will the stone hit the ground?

Ans. The time taken by the freely falling stone to reach the ground is given by,

$$t = \sqrt{\frac{2h}{g}}$$

In both the cases, the stone will fall through the same height as it is falling when the railway carriage is stationary. Hence, the stone will reach the ground after 2 seconds.

51. Is the speed of the projectile zero at the highest point of its trajectory, When a projectile is launched at an angle to the horizontal.

Ans. No, at its highest point of the trajectory of a projectile, the vertical component of the initial speed is zero, while the horizontal component of the initial speed remains unchanged. Thus, at the highest point of the trajectory, the speed of the projectile is equal to the horizontal component of the initial speed.

52. What will be the effect on horizontal range of a projectile when its initial velocity is doubled and keeping angle of projection same? **[Delhi Gov. QB 2022]**

Ans.

$$R = \frac{u^2 \sin 2\theta}{g} = R \propto u^2$$

Range becomes four times.

53. If the magnitude of two vectors is kept unchanged and the angle between them is changed, then what will be the effect on their resultant vector?

Ans. If their magnitude is same, but angle is changed of two vectors. The resultant vector's magnitude and direction will change.

54. What is the average value of acceleration vector in uniform circular motion over one cycle?

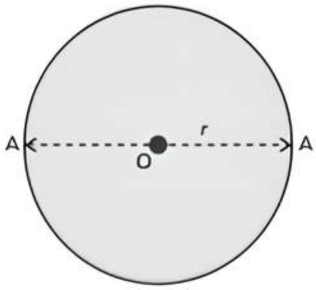
Ans. The average acceleration vector will be a null vector because the displacement is a null vector in one complete round, which implies that the average velocity is also a null vector, which further implies that the average acceleration will be a null vector for a particle moving in a uniform circular motion.

55. A bob suspended from a room's ceiling by a cord is set into oscillations. If the rope is pulled when the bob is at its mean position, what is the bob's trajectory? **[Diksha]**

Ans. When the bob is at its mean position, it has velocity parallel to horizontal direction. The bob is also acted by a force of gravity in the downward direction. This situation is equivalent to a projectile thrown horizontally from the top of the tower. So, when the string is cut in this position, the bob will follow the parabolic path.

56. Two girls run from point A to point B on the circular tracks. One runs along the tracks and the other runs directly along the diameter of the track. Calculate the ratio of magnitude of their displacement.

Ans. As the initial and final positions of both the girls are same, so the magnitude of their displacement are equal *i.e.*, equal to the radius of a circular track.



Then, the ratio of magnitude of displacement is 1.

SHORT ANSWER Type-I Questions (SA-I)

[**2** marks]

57. When a block is released from the top, it slides down a smooth inclined plane, while another ball falls freely from the same position. Which of them will fall to the ground first?

Ans. Acceleration of the freely falling block is g while that of the block sliding down the smooth inclined plane is where θ is the angle of inclination. On the other hand, the acceleration of the freely falling body is g. Since, $g > g \sin \theta$, so the block falling freely will reach the ground earlier.

58. **A particle is moving in a straight line. It travels a total distance of 4 m. A student says that the displacement of the particle is (4 m) or (–4 m) $\hat{i}$. Another student B says that the displacement of particle is zero. Do you agree with the statement made by both the students? Justify your answer.**

Ans. If a particle moves along positive x-axis and starts its journey from the origin to travel a total distance of 4 m along positive x-axis, the displacement of particle = (4 m) $\hat{i}$.

On the other hand, if a particle starts its journey from origin and travels a distance of 4 m along negative x-axis then displacement of particle =(–4 m) $\hat{i}$.

If a particle starts its journey from origin and travels 4 m towards from origin, then displacement = (4 m) $\hat{i}$ + (–4 m) $\hat{i}$ = 0

Thus, statements made by both students are correct.

59. **Can there be two vectors where the resultant is equal to either of them? [NCERT Exemplar]**

Ans. Yes, when two vectors of same magnitude inclined at an angle of 120°, then their resultant is equal to either of them.

Let x be the magnitude of each of two vectors which make an angle of 120° with other than their resultants is:

$$R = \sqrt{x^2 + x^2 + 2x \cdot x\cos 120°}$$

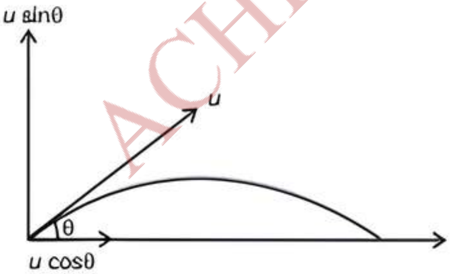
$$= \sqrt{2x^2 - x^2}$$

$$= \sqrt{x^2} = x$$

60. **Show that there are two values of time for which a projectile is at the same height. Also show that the sum of these two times is equal to the time of flight.**

[Delhi Gov. QB 2022]

Ans.



From figure, $u_y = u\sin\theta$, $a_y = -g$

y-direction : $S_y = u_y t + \frac{1}{2} a_y t^2$

$\therefore$ $h = (u\sin\theta)t - \frac{gt^2}{2}$

$\Rightarrow$ $t^2 - \left(\frac{2u}{g}\sin\theta\right)t + \frac{2h}{g} = 0$ ...(i)

Equation (i) has two roots t_1 and t_2, thus there are two values of time for which a projectile is at the same height.

Sum of roots of (i) $t_1 + t_2 = \frac{-b}{a} = \frac{2u\sin\theta}{g}$

= T(Time of flight)

61. **Can a vector vary with time? [Diksha]**

Ans. Yes, a vector can vary in time. For example, when a particle moves, its position vector continuously changes with time.

62. **The rotation of a body can be specified by the direction of the axis of rotation and the angle of rotation about the axis. Does that make any rotation a vector?**

Ans. Rotation, which is also called angular displacement, does not add like vectors. Two successive rotations are ϕ_1 and ϕ_2, that is,

$$\phi_1 + \phi_2 \neq \phi_2 + \phi_1.$$

Thus, finite rotations do not obey the commutative law of vector addition and so they are not vectors.

As rotations are made smaller, the result of operation $\phi_1 + \phi_2$ approaches that of operation $\phi_1 + \phi_2$.

If they are made infinitesimal, then the order of addition no longer affects the result ($d\phi_1 + d\phi_2 = d\phi_2 + d\phi_1$).

Hence, infinitesimal rotations are vector.

63. **Can the flight of a bird be an example of composition of vector?**

Ans. Yes, when a bird has to fly, it preens air with its two wings along the oblique directions. The resultant of the reactions on the two wings of the bird enables it to fly up in the sky.

64. **When the components of a vector $\vec{A}$ along the direction of $\vec{B}$ are zero. Then what do the two vectors depict?**

Ans. Consider two vectors $\vec{A}$ and $\vec{B}$ such that the angle between them is θ. The component of vector $\vec{A}$ along the direction of $\vec{B}$ is A cos θ.

Where A cos θ = 0, which is possible only when θ = 90°. Thus, vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

65. **A motorboat is racing towards north at 25 km/h and the water current in that region is 10 km/h in the direction of 60° east of south. Find the resultant velocity of the boat. [Delhi Gov. SQP 2022]**

Ans. Given,

Velocity of the water current v_c = 10 km/s
Velocity of the motorboat v_b = 25 km/s

Angle between north and south east is 120°

Resultant velocity

$$v_R = \sqrt{v_b^2 + v_c^2 + 2v_b v_c \cos 120°}$$

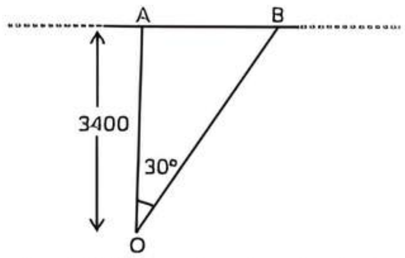
$$= \sqrt{(25)^2 + (10)^2 + 2(25)(10)\left(\frac{-1}{2}\right)}$$

$$= 22 \text{ km/h}^{-1}$$

The resultant velocity of the boat is 22 km/h^{-1}

66. If the angle subtended at a ground observation point by the aircraft positions 10 s apart is 30°, the aircraft is flying at a height of 3,400 m above the ground. What is the plane's top speed? [NCERT Exemplar]

Ans.



OA is the height of the aircraft. In 10 s the aircraft covers a distance AB.

In triangle AOB,

$$\tan\theta = \frac{AB}{OA}$$

or $AB = OA \times \tan\theta$

By substituting the given values in the above expression, we get

$AB = 3400 \times \tan(15°)$
$= 911$ m

The distance covered by aircraft in 10 s is,

$OB = 2AB = 2 \times 911$
$= 1822$ m

The speed of the aircraft is given as,

$$v = \frac{\text{distance}}{\text{time}} = \frac{OB}{t}$$

By substituting the given values in the above expression, we get

$$v = \frac{1822}{10} = 182.2 \text{ ms}^{-1}$$

Thus, the speed of the aircraft is 182.2 ms^{-1}.

67. Standing on the edge of a cliff, a person throws a ball straight up at a specific speed and then throws another ball straight down at the same speed. If air resistance is minimal, which is the case, When the ball hits the ground below, which one has the faster speed?

Ans. If air resistance is negligible, both balls will strike the ground with the same speed. It is because the ball thrown upwards reaches its maximum height with zero speed and again falls down with an increment of 10 m/s in the speed each second. Thus, it reaches the point from where it was thrown up with the same initial speed of throw. In this way, both the balls, in a way, are thrown downward from the same point with the same initial speed. Hence, both of them will strike the ground with the same speed.

68. Consider a parabolic path taken by a batted baseball on a day when the sun is directly overhead. What is the relationship between the speed of a ten ball's shadow across the field and the horizontal component of velocity of the ball?

Ans. The speed of ball shadows across the field with the horizontal component of velocity of ball have the same velocities because the shadow will move along with the ball.

Related Theory

→ *If an object follows a parabolic path, then at each instant there will be two components of velocity – Horizontal and vertical components and shadow is just the reflection. A ball follows a parabolic path, movement of the ball shadow isn't affected by friction and also there will be no loss. So, the shadow of the ball will move along the ball, when the sun is overhead as it will not be considered by any other factor.*

69. The sum of the magnitudes of two forces acting at a point is 18 N and the magnitude of their resultant is 12 N. If the resultant makes an angle of 90° with the force of smaller magnitude, what are the magnitudes of the two forces? [Diksha]

Ans. Let the magnitude of smaller force is P, the magnitude of larger force is Q and resultant force is R.

As the resultant force makes 90° with the smaller force P then Q

Thus, $Q^2 - P^2 = R^2$

$Q^2 - P^2 = (12)^2$

$Q^2 - P^2 = 144$ ...(i)

Sum of magnitudes,

$P + Q = 18$

$Q = 18 - P$ ...(ii)

On substituting the value for eqn (ii) in (i), we get

$(18 - P)^2 - P^2 = 144$

$P = 5$ N

Then, $Q = 18 - 5$

$= 13$ N

Thus, the magnitude of the forces are 5 N and 13 N.

70. **Establish the following vector inequalities:**

(A) $|\vec{a}+\vec{b}| \le |\vec{a}|+|\vec{b}|$ **(B)** $|\vec{a}+\vec{b}| \ge |\vec{a}|+|\vec{b}|$

[Delhi Gov. QB 2022]

Ans. We know,

$$|a+b| = \sqrt{(a+b)(a+b)}$$
$$= \sqrt{a^2+b^2+2ab\cos\theta}$$
$$= \sqrt{|a|^2+|b|^2+2|a||b|\cos\theta}$$

For maximum cos θ = + 1

For minimum cos θ = – 1

e.g.

$$\sqrt{|a|^2+|b|^2+2|a||b|(-1)} \le |a+b| \le \sqrt{|a|^2+|b|^2+2|a||b|(+1)}$$

$$\therefore \sqrt{(|a|-|b|)^2} \le |a+b| \le \sqrt{(|a|+|b|)^2}$$

(|a| + |b|) > 0 for all real number

So, $\sqrt{(|a|-|b|)^2} = |a|+|b|$

But, (|a| – |b|) may be positive or negative

So, $\sqrt{(|a|-|b|)^2} = |a|-|b|$

Hence,

$|a| - |b| \le |a+b| \le |a| + |b|$

(A) $|a + b| \le |a| + |b|$, this equality applies. If '*a*' and '*b*' are acting in same correction. *e.g.*, angle between them = 0

(B) When, $|a| > |b|$ then, $|a + b| > |a| - |b|$, this equality applies. If *a* and *b* are in opposite directions and magnitude of '*a*' greater than magnitude of '*b*'.

SHORT ANSWER Type-II Questions (SA-II)

[**3** marks]

71. **Prove that the maximum horizontal range is four time the maximum height attained by the projectile, when fired at an inclination so as to have maximum range.**

[Delhi Gov. QB 2022]

Ans. The horizontal range is maximum when the angle of projection is 45°.

So, $$R = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{u^2 \sin(2\times45^\circ)}{g} = \frac{u^2}{g} \quad \text{...(i)}$$

Maximum height for angle of projection 45° is,

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$= \frac{u^2 \sin^2 45^\circ}{2g} = \frac{u^2}{4g} \quad \text{...(ii)}$$

Therefore, from equation (i) and (ii),

R = 4H

That is, range is 4 times the maximum height attained by a projectile.

72. **If $\vec{a}, \vec{b}$ and $\vec{c}$ are represented by three sides of triangle taken in same order.**

Proove that: $\vec{a} + \vec{b} + \vec{c} = 0$.

[Delhi Gov. QB 2022]

Ans. Let $\vec{AB} = \vec{a}$, $\vec{BC} = \vec{b}$, $\vec{CA} = \vec{c}$

Then, by triangle law of vector addition

$\vec{AB} + \vec{BC} + \vec{AC}$

$$\vec{AB} + \vec{BC} - \vec{AC} = 0$$

$$\vec{AB} + \vec{BC} + \vec{CA} = 0 \qquad (\vec{AB} = -\vec{BA})$$

$$\vec{a} + \vec{b} + \vec{c} = 0$$

73. **A sling has a small cradle in the middle of two cords. A projectile is placed in the pouch. By its double pendulum kinetics, the slings enable stone to be thrown much farther than they could be by hand alone. Can this working of sling justifies the parallelogram law of addition of vectors?**

Ans. A sling consists of a Y-shaped wooden or metallic frame, to which a rubber band is attached. When a stone held at the point on the rubber band is pulled, the tensions T_1 and T_2 are produced in two segments of the rubber band now. According to the parallelogram Laws of forces, resultant T of the tensions T_1 and T_2 acts on the stone. As the stone is released, it moves under the action of the resultant tension T in forward direction with a large speed.

74. **A man moving in the rain holds his umbrella inclined to the vertical even though the raindrops are falling vertically downwards. Why?** **[NCERT Exemplar]**

Ans. When the man moves in the rain, falling vertically downwards, the rain drops appear to fall in a direction inclined to the vertical. So, to protect himself from the rain, he holds the umbrella inclined to the vertical in the direction of apparent velocity to the rain with respect to himself.

75. **A bullet fired at an angle of 30° with the horizontal hits the ground 3 km away. By adjusting the angle of projection, can one hope to hit the target 5 km away? Assume that the muzzle speed to be fixed and neglect air resistance. [Delhi Gov. QB 2022]**

Ans. Given Range, R = 3 km

Angle of projection = 30

Acceleration due to gravity, $g = 9.8\ m/s^2$

Horizontal range for the projection velocity u_0, is given by the relation:

$$R = \frac{u_0^2 \sin 2\theta}{g}$$

$$3 = \frac{u_0^2 \sin 60°}{g}$$

$$\frac{u_0^2}{g} = 2\sqrt{3} \quad \text{...(i)}$$

The maximum range (R_{max}) is achieved by the bullet when it is fired at an angle of 45 with the horizontal, that is,

$$R_{max} = \frac{u_0^2}{g} \quad \text{...(ii)}$$

On comparing equations (i) and (ii), we get:

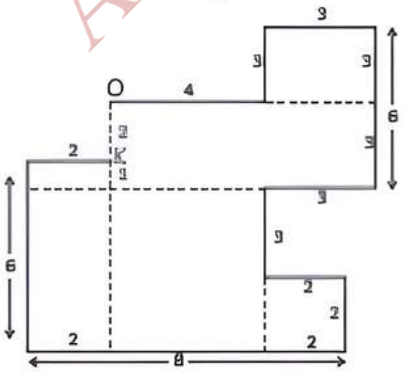
$$R_{max} = 2 \times 1.732$$

$$= 3.46\ \text{km}$$

Hence, the bullet will not hit a target 5 km away.

76. **A man follows the following route-from his house, he travelled four blocks east, three blocks north, three blocks east, six blocks south, three blocks west, three blocks south, two blocks east, two blocks south, eight blocks west, six blocks north and finally two blocks east. If all the blocks are of the same length, how far and in what direction will he be from his house?**

Ans. Starting from the point O. The house of the man, the route followed by him as shown in figure. The man finally reaches the end point K.



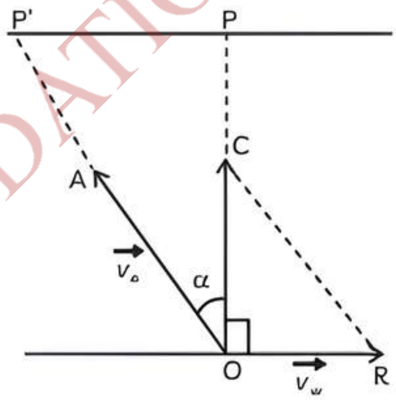
It follows that the man is now two blocks south of his house and his total displacement is also two blocks south.

77. **A river is flowing from west to east at a speed of 5 m per min. A man on the south bank of the river capable of swimming at 10 ms^{-1} in still water, wants to swim across the river in the shortest time. In what direction should he swim?**

Ans. Let the man start swimming from point O to swim across the river in shortest time. If he reaches the point P on the other bank directly opposite to the starting point O. Then the men should strike in a direction making an angle with OP, so after being drifted by water current, he swims along OP as,

The resultant of velocity of man in still water

$\vec{v}_M = (\vec{OA})$ and velocity of water $\vec{v}_w = (\vec{OB})$ is along OP is given by, $\vec{v} = (\vec{OC})$



In right-angled triangle, OAC,

$$\text{Sin } \alpha = \frac{AC}{OA} = \frac{OB}{OA}$$

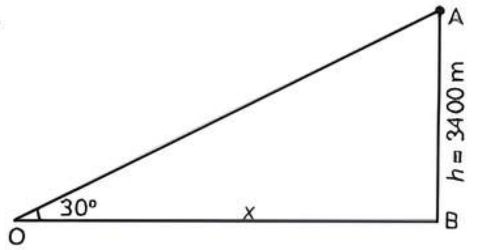
$$= \frac{v_w}{v_A} = \frac{5}{10} = \frac{1}{2}$$

Or $\alpha = 30°$

Therefore, the man should start swimming at an angle 30° + 90° = 120° with the bank of the river (or with the direction of flow of the river).

78. **An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft position 10 second apart is 30°, what is the speed of the aircraft? [Delhi Gov. QB 2022]**

Ans.



In right angled triangle OAB. $\tan 30° = \frac{h}{x}$

We get. $x = \frac{h}{\tan 30°}$

$\therefore \quad = 3400\sqrt{3}$

$= 5889$ m

Time taken. $t = 10.0$ s

Thus speed of the aircraft.

$$v = \frac{x}{t}$$

$$\therefore \quad v = \frac{5889}{10.0} = 588.9 \text{ m/s}$$

79. A body is simultaneously given two velocities on 30 ms^{-1} due east and other 40 ms^{-1} due north. Find the resultant velocity.

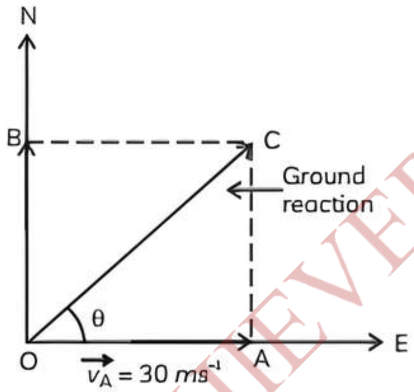
Ans. Given. $v_A = 30$ ms^{-1} due east. $v_B = 40$ ms^{-1} due north

Its resultant:

$$v = \sqrt{v_A{}^2 + v_B{}^2}$$

$$= \sqrt{30^2 + 40^2} = 50 \text{ ms}^{-1} v_B$$

$$v_B = 40 \text{ ms}^{-1}$$



The velocity v makes θ with east direction.

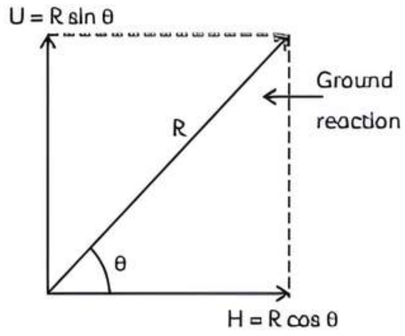
So,

$$\tan\theta = \frac{CA}{OA} = \frac{40}{30} = 1.33$$

$$\theta = \tan^{-1}(1.33) = 53°$$

80. Can the walk of a man be an example of the resolution of vector? [Diksha]

Ans. While walking a person preens the ground with his feet slightly slanted in the background direction. Ground exerts upon him an equal and opposite reaction R. Its horizontal component is H = R cos θ enables the person to move forward while the vertical component U = R sin θ balances his weight.



81. (A) If $\hat{i}$ and $\hat{j}$ are unit vectors along x and y axis respectively then what is magnitude and direction of $(\hat{i}+\hat{j})$ and $(\hat{i}-\hat{j})$?

(B) Find the components of vector $\vec{a} = 2\hat{i} + 3\hat{j}$ along the directors of vectors $(\hat{i}+\hat{j})$ and $(\hat{i}-\hat{j})$. [Delhi Gov. QB 2022]

Ans. For $\hat{i} + \hat{j}$

Magnitude $= \sqrt{1^2 + 1^2} = \sqrt{2}$

For Direction $= \tan^{-1}\left(\frac{1}{1}\right) = +45°$ to the x-axis

$\hat{i} - \hat{j}$

Magnitude $= \sqrt{1^2 + (-1)^2} = \sqrt{2}$

Direction $= \tan^{-1}\left(-\frac{1}{1}\right) = -45°$ to the x-axis

Let. $\vec{a} = 2\hat{i} + 3\hat{j}$, $\vec{b} = \hat{i} + \hat{j}$, $\vec{c} = \hat{i} - \hat{j}$ to get component of $\vec{a}$ along directon of $\vec{b}$.

We need ∠ between the two

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{2+3}{\sqrt{3}+\sqrt{2}} = \frac{5}{\sqrt{26}}$$

So, component of a along b is $\frac{5}{\sqrt{26}}(2\hat{i} + 3\hat{j})$

Similarly, for component of a along direction of c.

$$\cos\theta = \frac{\vec{a}\cdot\vec{c}}{|\vec{a}||\vec{c}|} = \frac{-1}{\sqrt{26}}$$

So, component $= \left(\frac{-1}{\sqrt{26}}\right)(2\hat{i} + 3\hat{j})$

82. Reader is an electronic system which measures the range and bearing of objects by transmitting an electromagnetic pulse at object then at what range will a radar set show a plane flying at 10 km above the ground and what distance of 18 km from the radar station?

Ans. A displacement of 18 km from the radius station and then another displacement of

10 km up gives the displacement of the plane from the radar station.

Therefore, range = magnitude of displacement of plane from radar station.

$$= \sqrt{18^2 + 10^2} = 20.6 \text{ km}$$

83. On a straight road, a police officer in a jeep is hunting a pickpocket. The jeep is travelling at maximum speed (assumed uniform). When a pickpocket rides on the motorbike of a waiting friend when the jeep is d kilometres away and the motorcycle accelerates at a constant rate of a, what will be the velocity of the jeep to catch the thief?

Ans. Suppose the pickpocket is caught at a time t after the motorcycle starts. The distance travelled by the motorcycle during this interval is, $s = \frac{1}{2}at^2$.

During this interval the jeep travel a distance $s + d = vt$.

So, by using both the equations we have:

$$\frac{1}{2}at^2 - vt + d = 0$$

Or $$t = \frac{v \pm \sqrt{v^2 - 2ad}}{a}$$

The pickpocket will be caught if t is real and positive. This will be possible if

$$v^2 \geq 2ad \text{ or } v \geq \sqrt{2ad}$$

84. A body is projected at an angle θ with the horizontal. Derive an expression for its horizontal range. Show that there are two angles θ_1 and θ_2 projections for the same horizontal range, such that $\theta_1 + \theta_2 = 90°$.

[Delhi Gov. QB 2022]

Ans. Let the two angles be $\theta_1 + \theta_2$ and velocity v,

Horizontal Range,

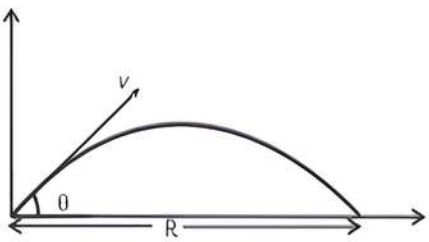
$$R = \frac{v^2 \sin(2\theta_1)}{g} = \frac{v^2 \sin(2\theta_2)}{g}$$

$$\Rightarrow \quad = \frac{v^2 \sin(180 - 2\theta_2)}{g}$$

Using $\sin(\pi - \theta)$

$$\Rightarrow \quad 2\theta_1 = 180° - 2\theta_2$$

$$\theta_1 + \theta_2 = 90°$$



Initial velocity v making angle θ

$$v_x = v\cos\theta$$

$$v_y = v\sin\theta$$

$$v = \sqrt{v_x^2 + v_y^2}$$

Horizontal range, R = velocity is x × time of flight

$$= v_x T$$

$$= v\cos\theta T \quad \text{...(i)}$$

Now, we know $v = u + at$

$$v_y = v_y(0) - gt$$

Acceleration = $-g$, motion upward and gravity downward

∴ − ve

So, at highest point

$$0 = v_y - gt$$

$$t = \frac{v\sin\theta}{g}$$

Total time of flight,

$$2t = \frac{2v\sin\theta}{g}$$

Now, $$R = \frac{v\cos\theta - 2v\sin\theta}{g}$$

$$R = \frac{v^2 \sin 2\theta}{g}$$

85. An anti-aircraft cannon is passed directly overhead by a fighter plane flying horizontally at 1.5 km altitude and 720 km/h. What angle should the gun be fired from in order for the muzzle-speed shell to impact the plane? To prevent being hit, what is the least altitude at which the pilot should fly the plane?

Ans. Height of the fighter plane

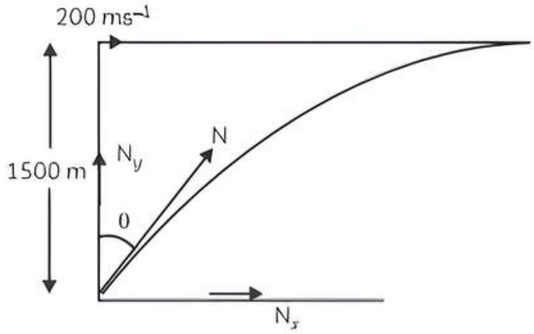
$$= 1.5 \text{ km} = 1500 \text{ m}$$

Speed of the fighter plane,

$$v = 720 \text{ km/h}$$

$$= 200 \text{ m/s}$$

Let θ be the angle with the vertical so that the shell hits the plane. The situation is shown in the given figure.



Muzzle velocity of the gun, $u = 600$ m/s

Time taken by the shell to hit the plane = t

Horizontal distance travelled by the shell = $u_x t$

Distance travelled by the plane = vt

The shell hits the plane. Hence, these two distances must be equal.

$$u_x t = vt$$

$$u \sin\theta = v$$

$$\sin\theta = \frac{v}{u}$$

$$= \frac{200}{600} = \frac{1}{3} = 0.33$$

$$\theta = \sin^{-1}(0.33) = 19.5°$$

In order to avoid being hit by the shell, the pilot must fly the plane at an altitude (H) higher than the maximum height achieved by the shell.

$$H = \frac{u^2 \sin^2(90-\theta)}{2g}$$

$$= \frac{600^2 \cos^2\theta}{2g}$$

$$= 18000 \times (0.943)^2$$

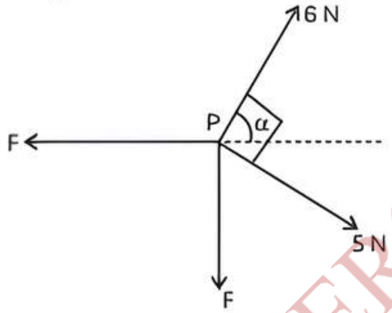
$$= 16006.42 \text{ m}$$

or $$= 16 \text{ km.}$$

LONG ANSWER Type Questions (LA)

[**4** & **5** marks]

86. A particle P is in equilibrium on a smooth horizontal table under the action of four horizontal forces of magnitude 6 N, 5 N, P (in N) and F (in N) acting in the direction as shown in figure. Find the values of α and F.



Ans. As the particle P is in equilibrium, the resultant of forces F and F is equal and opposite to the resultant of forces of 6 N and 5 N.

$$F = \sqrt{F^2 + F^2}$$

$$= \sqrt{6^2 + 5^2}$$

$$F = 7.81 \text{ N}$$

The resultant of the forces of F and F is inclined at 45° with the horizontal. Since the resultant of forces of 6 N and 5 N is directed opposite to that of forces F and F, the resultant of the forces of 6 N and 5 N makes an angle of 45° with the horizontal or an angle of (α – 45°) with the forces of 6 N. As the forces of 6 N and 5 N are perpendicular to each other.

$$\tan(\alpha - 45°) = \frac{5}{6} = 0.83333$$

or $$\alpha - 45° = 39.8°$$

$$\alpha = 84.8°$$

87. A projectile is fired at an angle θ. Find expression for:

(A) Maximum height

(B) Total time of flight

(C) Horizontal range [Delhi Gov. SQP 2022]

Ans. (A) Let H be the maximum height reached by the projectile in time t_1 for vertical motion.

The initial velocity = $u \sin\theta$

The final velocity = 0

Acceleration = $-g$

∴ using, $$v^2 = u^2 + 2as$$

$$0 = u^2 \sin 2\theta - 2gH$$

$$2gH = u^2 \sin 2\theta$$

$$H = \frac{u^2 \sin 2\theta}{2g}$$

(B) Let t_1 be the time taken by the projectile to reach the maximum height H.

For vertical motion.

Initial velocity = $u\sin\theta$

Final velocity at the maximum height = 0

Acceleration $a = -g$

Using the equation

$$v = u + at_1$$

$$0 = u\sin\theta - gt_1$$

$$gt_1 = u\sin\theta$$

$$t_1 = \frac{u\sin\theta}{g}$$

Let t_2 be the time of descent.

But $t_1 = t_2$

i.e., time of ascent = time of descent.

∴ Time of flight

$$T = t_1 + t_2 = 2t_1$$

$$\therefore \quad T = \frac{2u\sin\theta}{g}$$

(C) Let R be the range of the projectile in a time T. this is covered by the projectile with a constant velocity $u\cos\theta$.

Range = horizontal component of velocity × time of flight

i.e., $R = u\cos\theta . T$

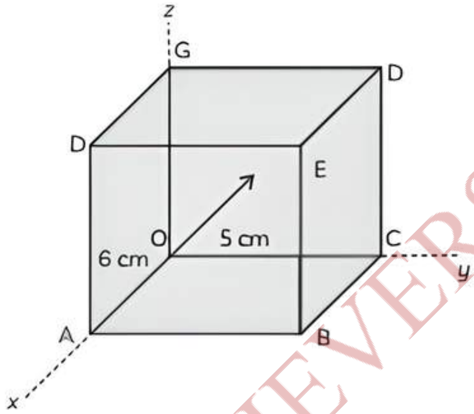
$$R = u\cos\theta . \frac{2u\sin\theta}{g}$$

$$R = \frac{u^2\sin 2\theta}{g}$$

$\therefore 2\sin\theta\cos\theta = \sin 2\theta$

88. A room has dimension 6 × 5 × 4 m. A fly starting at one corner, ends up at a diametrically opposite corner. What is the magnitude of displacement? Could the length of its path be less than or greater than or equal to the distance? If the fly does not fly but crawls, what is the length of the shortest path it can take?

Ans.



The side of the rooms are:

$\vec{OA}$ = 6 cm (along x-axis)

$\vec{OC}$ = 5 cm (along y-axis)

$\vec{OG}$ = 4 cm (along z-axis)

Suppose fly starts from corner O and flies at the diametrically opposite corner E.

Then displacement of fly is

$$\vec{OE} = \vec{OA} + \vec{AB} + \vec{BE}$$

$$= 6\hat{i} + 5\hat{j} + 4\hat{k}$$

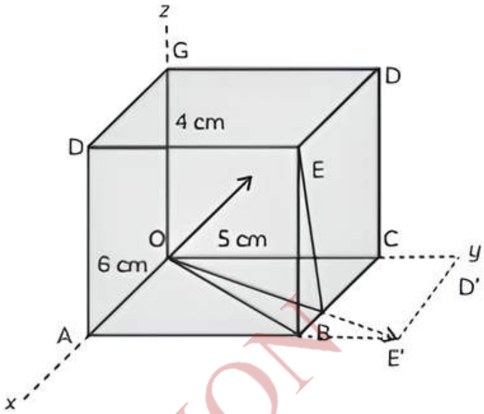
And magnitude is

$$|\vec{OE}| = \sqrt{6^2 + 5^2 + 4^2}$$

$$= \sqrt{77} = 8.77 \text{ m}$$

As displacement between two points is the shortest path, the length of any path followed by the fly will be greater than or at least equal to 8.77 m.

In order to find the length of shortest path along which fly can reach corner E by crawling. Imagine the wall BCDE.



has laid down as BCD'E besides the floor OABC of the room.

The shortest path between the corners OE, the fly has to crawl along OK on the floor and then along KE on the wall BCDE.

The length of shortest path

$$\vec{OE'} = \sqrt{OA^2 + AE^2}$$

$$\vec{OA} = 6\hat{i} \text{ and } \vec{AE'} = (5+4)\hat{j} = 9\hat{j}$$

$$\vec{OE'} = \sqrt{6^2 + 9^2} = \sqrt{117} = 10.82$$

89. If R is the horizontal range for θ inclination and h is the maximum height attained by the projectile, show that the maximum range is given by $\frac{R^2}{8h} + 2h$.

[Delhi Gov. QB 2022]

Ans.

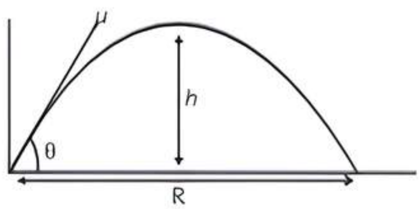


Fig. 1

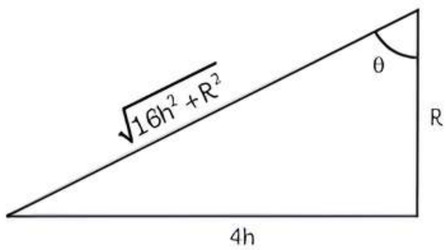


Fig. 2

$H_{max} = h$

Maximum height at the projectile

$$h = \frac{u^2 \sin^2\theta}{2g} \quad ...(i)$$

$$\text{Range } R = \frac{u^2 \sin 2\theta}{g} \quad ...(ii)$$

Dividing equation (i) and (ii)

$$\frac{h}{R} = \frac{\sin^2\theta}{2\sin 2\theta}$$

$$= \frac{\sin^2\theta}{2 \times 2\sin\theta\cos\theta} = \frac{\tan\theta}{4}$$

$$\Rightarrow \quad \frac{4h}{R} = \tan\theta \quad ...(iii)$$

From equation (iii) [Refer Fig. 2]

$$\sin\theta = \frac{4h}{\sqrt{16h^2 + R^2}} \quad ...(iv)$$

Step 2: Calculations

For maximum range $\theta = 45°$

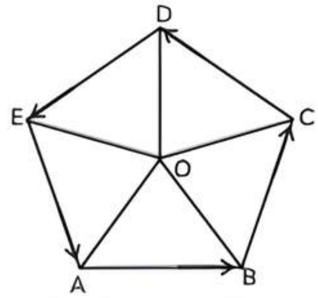
$$\Rightarrow \quad R_{max} = \frac{u^2}{g} \quad ...(v)$$

From equation (i), (iv) and (v)

$$h = \frac{R_{max} 16h^2}{2(16h^2 + R^2)}$$

$$\Rightarrow \quad R_{max} = 2h + \frac{R^2}{8h}$$

90. **Five vectors are represented by the side of a closed pentagon taken in same vector order. Find their resultant.** **[Diksha]**



Ans. $\vec{AB} = \vec{AO} + \vec{OB} = -\vec{OA} + \vec{OB}$

$\vec{BC} = \vec{BO} + \vec{OC} = -\vec{OB} + \vec{OC}$

$\vec{CD} = \vec{CO} + \vec{OD}$

$\vec{DE} = \vec{DO} + \vec{OE}$

$\vec{EA} = \vec{EO} + \vec{OA}$

Adding all

$\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} + \vec{EA} = 0$

NUMERICAL Type Questions

91. **An aeroplane takes off at an angle of 60° to the horizontal. If muzzle velocity of plane is 200 Kmh^{-1}. Calculate horizontal and vertical components.** **(2m)**

Ans. Given, $v = 200$ km h^{-1} and $\theta = 60°$

Horizontal component

$= v\cos\theta$

$= 200 \cos 60°$

$= 200 \times \frac{1}{2} = 100$ km h^{-1}

Vertical component

$= v\sin\theta$

$= 200 \sin 60°$

$= 200 \times \frac{\sqrt{3}}{2}$

$= 100\sqrt{3}$ km h^{-1}

92. **A body is projected upwards with a velocity u. It passes through a certain point above the ground after t_1 second. Then at what time after which the body passes through the same point during the return journey?** **(2m)**

Ans. As we know, $t_{asc} = \frac{u}{g}$

$\therefore$ Time for the upward journey

$$= (t_{asc} - t_1) = \left[\frac{u}{g} - t_1\right]$$

$$\text{Total time} = 2\left[\frac{u}{g} - t_1\right]$$

93. **A stone is dropped from the top of a tower of height h. After 1 second another stone is dropped from the balcony 20 m below the top of the tower. Both reach the bottom simultaneously. What is the value of h?** **[NCERT Exemplar](2m)**

Ans.

$$h = \frac{1}{2}gt^2$$

$$h - 20 = \frac{1}{2}g(t-1)^2$$

$$\frac{1}{2}gt^2 - 20 = \frac{1}{2}gt^2 - gt + \frac{g}{2}$$

$$gt = 20 + \frac{g}{2}$$

$$10t = 20 + 5 \quad [\because g = 10 \text{ m/s}^2]$$

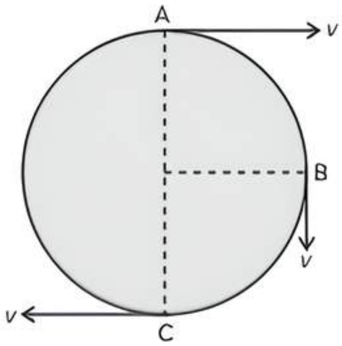
$$\Rightarrow \quad t = 2.5 \text{ sec}$$

$$h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (2.5)^2$$

$$h = 5 \times 62.5 = 31.25 \text{ m}$$

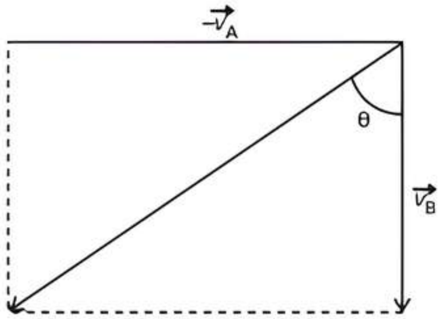
94. **A car is moving around a circular track with a constant speed of v of 20 ms^{-1}. At different times, the car is at A, B and C respectively. Find the velocity from A to C and from A to B.** **(3m)**

Ans. Change in velocity, car move from A to C



$\Delta \vec{v}_{CA} = \vec{v}_C - \vec{v}_A = 20-(-20) = 40\ ms^{-1}$

$\Delta \vec{v}_{CA} = 40\ ms^{-1}$ in direction of $\vec{BC}$



Change in velocity as car moves from A to B.

$\Delta \vec{v}_{BA} = \vec{AB} - \vec{v}_A = \vec{v}_B + (-\vec{v}_A) = 20-(-20) = 40\ ms^{-1}$

$\Delta \vec{v}_{BA} = \sqrt{20^2+20^2} = \sqrt{800}\ ms^{-1} = 28.28\ ms^{-1}$

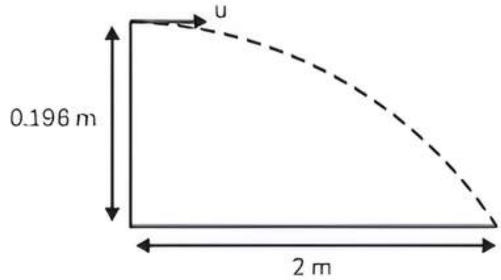
Also, $\tan\theta = \dfrac{20}{20} = 1$

$\Rightarrow \theta = 45°$

This is the required direction of change in velocity.

95. **Goli, a game played with small glass balls is a popular game in Indian communities. The goli of one player is located 2.0 meters apart from the goli of the other player. This second player must project his goli by keeping his left hand's thumb at the goli's location, holding the goli between his two middle fingers, and throwing it. The second player wins if the projected goli hits the first player's goli. If the goli is projected horizontally from a height of 19.6 cm above the ground, what speed should it be projected at so that it hits the stationary goli directly?** **(3m)**

Ans. The goli move like a projectile.



Here, $h = 0.196$ m

Horizontal distance, $x = 2$ m

Acceleration, $g = 9.8\ m/s^2$

Time to reach the ground

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 0.196}{9.8}} = 0.2 \text{ sec}$$

Let horizontal velocity with which it is projected be u m/s

$$\therefore \quad x = ut \text{ or } u = \frac{x}{t}$$

$$= \frac{2}{0.2} = 10 \text{ m/s.}$$

96. **A cyclist is travelling at a speed of 27 kilometres per hour. As he approaches a circular turn on a road with a radius of 80 metres, he hits the brakes and slows down at a steady pace of 0.50 metres per second. What is the magnitude and direction of the cyclist's net acceleration during a circular turn?** **(5m)**

Ans. 54.46° with the direction of velocity.

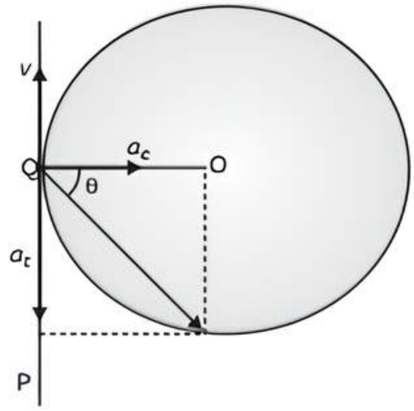
Speed of the cyclist, $v = 27$ km/h $= 7.5$ m/s

Radius of the circular turn, $r = 80$ m

Centripetal acceleration is given as:

$$a_c = \frac{v^2}{r} = \frac{7.5^2}{80} = 0.7\ ms^{-2}.$$

The situation is shown in the given figure:



Suppose the cyclist begins cycling from point P and moves toward point Q. At point Q, he applies the brakes and decelerates at the rate of 0.5 m/s^2.

This acceleration is along the tangent at Q and opposite to the direction of motion of the cyclist. The resultant acceleration a is given by:

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{(0.7)^2 + (0.5)^2} = \sqrt{0.74}$$

$$= 0.86\ m/s^2$$

Since the angle between a_c and a_t is

$$\tan\theta = \frac{a_c}{a_t} = \frac{0.7}{0.5} = 1.4$$

$$\theta = \tan^{-1}(1.4) = 54.46°$$

97. The greatest height to which a boy throws a ball is 50 m. What will be the greatest distance along the horizontal up to which the boy can throw the ball with the same speed? [NCERT Exemplar](5m)

Ans. It is given that the boy can throw a stone to a maximum height of 50 m.

To throw a stone to maximum height, the boy has to throw the stone vertically upwards.

Third equation of motion: $v^2 = u^2 + 2as$

Final velocity of the ball, $v = 0$

Acceleration of the ball

= -Acceleration due to gravity = $-g$

Displacement of the ball, $s = 50$ m

Putting all these values to the third equation of motion, we get,

$$0 = u^2 - 2g \times 50$$

$$\Rightarrow \quad 0 = u^2 - 2g \times 50 \quad \text{– (i)}$$

$$u^2 = 100g$$

Maximum range,

$$R_{max} = \frac{u^2}{g}$$

Putting values in above equation, we get

$$R_{max} = \frac{100g}{g} = 100\ m$$

Now, maximum height for maximum range,

$$h_{max} = \frac{u^2}{2g}\sin\theta$$

Putting values in above equation, we get

$$3\theta = 45°,$$

for maximum range

$$h_{max} = \frac{50}{2} = 25\ m$$

Hence, Maximum range of the ball is 100 m and greatest height for maximum range is 25 m.