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	Subject : PHYSICS	
	Class : XI	

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1   Deduce the dimensional formula for R, using ideal gas equation $PV = nRT$ **1**

Ans : $PV = nRT$

$$R = \frac{PV}{n\,T}$$

(n is a number of molecules)

$$R = \frac{[ML^{-1}T^{-2}]L^3}{K}$$

$$= [ML^2 T^{-2} K^{-1}]$$

2   Find degree of freedom of a monoatomic gas.   **1**

Ans : No. of degree of freedom for monoatomic gas,

$$dof = 3N - K$$

where N is the number of atoms and K is the number of constraints

$$N = 1, K = 0$$
$$dof = 3 \times 1 - 0$$
$$dof = 3$$

Therefore, degree of freedom of a monoatomic gas is 3.

3   What would be the effect on the *rms* velocity of gas molecules if the temperature of the gas is increased by a factor of 4 ?   **1**

Ans : $\because$ $v_{rms} = \sqrt{T}$

So, v_{rms} is doubled.

4   If a molecule having N atoms has k number of constraints, how many degrees of freedom does the gas possess ?   **1**

Ans : Degree of freedom $f = 3\,N - K$.

5   If there are f degrees of freedom with n moles of a gas, find the internal energy possessed at a temperature T.   **1**

Ans : For 1 mole with f degrees of freedom
Internal energy,

$$U = 1 \times C_v \times T = \frac{f}{2}RT$$

For n moles,

$$U = n \times C_v \times T = \frac{nf}{2}RT$$

6   What is the law of equi-partition of energy ?   1

Ans :

According to law of equipartition of energy, for any dynamical system in thermal equilibrium the total energy is distributed equally amongst all the degrees of freedom and the energy associated with each molecule per degree of freedom is $\frac{1}{2}K_BT$, where K_B is Boltzmann constant and T is temperature of the system.

7   The energy of a given sample of an ideal gas depends only on its   1
(a) volume
(b) pressure
(c) temperature
(d) mass

Ans :

(c)

The energy of a given sample of an ideal gas depends only on temperature as average energy/molecule/degree of freedom = $\frac{1}{2}k_BT$

8   If C_P, C_V are molar specific heats of a solid and R is universal gas constant,   1
then
(a) $C_P - C_V = R$
(b) $C_P - C_V = 0$
(c) $C_P - C_V$ is negative
(d) $(C_P - C_V) << R$

Ans : (d)

In case of solids, $C_P = C_V$

$\therefore C_P - C_V << R$

9   For a gas, $R/C_V = 0.67$. The gas is made up of molecules, which are   1
(a) monoatomic
(b) diatomic
(c) triatomic

(d) polyatomic

Ans :
(a) $\dfrac{R}{C_V} = 0.67 = \dfrac{2}{3}$ The gas must be monoatomic.

$C_v = \dfrac{3}{2}R$

10 The pressure P of a gas and its mean K.E. per unit volume are related as   1

(a) $P = \dfrac{1}{2}E$ (b) $P = E$

(c) $P = \dfrac{3}{2}E$ (d) $P = \dfrac{2}{3}E$

Ans : (d)

$$P = \frac{1}{3}\rho C^2$$

$$\text{Mean K.E./volume} = E = \frac{1}{2}\rho C^2$$

$$\therefore \quad P = \frac{1}{3}\rho C^2 = \frac{2}{3}\left(\frac{1}{2}\rho C^2\right) = \frac{2}{3}E$$

11 A perfect gas at 27 °C is heated at constant pressure so as to double its volume.   1
The temperature of the gas will be:
(a) 300 °C
(b) 54 °C
(c) 600 °C
(d) 327 °C

Ans : (d)

According to Charle's law, when P is constant, $T \propto V$
As V is doubled, T becomes twice i.e.,
$T = 2 \times (17 + 273)$ K $= 600$ K
$= 600 - 273 = 327$ °C

12 The average K.E. of a gas molecule at 25° C is 6.21×10^{-21} J. Its average K.E.   1
at 227° will be
(a) 52.2×10^{-21} J
(b) 5.21×10^{-21} J
(c) 10.35×10^{-21} J
(d) 11.35×10^{-21} J

Ans : (c)

$$E_2 = E_1 \sqrt{\frac{T_2}{T_1}}$$

$$= 6.21 \times 10^{-21} \frac{(263 + 227)}{(273 + 27)}$$

$$= 10.35 \times 10^{-21} \text{ J}$$

13 Oxygen and hydrogen gases are at same temperature and pressure. And the oxygen molecule has 16 times the mass of hydrogen molecule. Then the ratio of their r.m.s. speed is: 1

(a) 2

(b) 1/4

(c) 4

(d) 16

Ans :
  (b) $\dfrac{C_{oxy}}{C_H} = \sqrt{\dfrac{m_H}{m_{oxy}}} = \sqrt{\dfrac{1}{16}} = \dfrac{1}{4}$

14 The molar specific heat at constant pressure of an ideal gas is $\left(\dfrac{7}{2}R\right)$. constant   1

pressure to that at constant volume is

(a) 9/7

(b) 7/5

(c) 5/7

(d) 8/7

Ans :

$$C_P = \frac{7}{2}R$$

(b) $C_V = C_P - R = \dfrac{7}{2}R - R = \dfrac{5}{2}R$

$\gamma = \dfrac{C_P}{C_V} = \dfrac{7/2P}{5/2R} = \dfrac{7}{5}$

15 A cubic vessel (with faces horizontal + vertical) contains an ideal gas at NTP. The vessel is being carried by a rocket which is moving at a speed of 500m s^{-1} in vertical direction. The pressure of the gas inside the vessel as observed by us on the ground   1

(a) remains the same because 500 ms^{-1} is very much smaller than vrms of the gas.

(b) remains the same because motion of the vessel as a whole does not affect the relative motion of the gas molecules and the walls.

(c) will increase by a factor equal to $[v^2_{rms} + (500)^2]/v^2$ where v_{rms} was the original mean square velocity of the gas.

(d) will be different on the top wall and bottom wall of the vessel.

Ans : *(b)*

As $P = \dfrac{nRT}{V}$, it (i.e., P) remains unaffected as n, R, T and V are constants. In fact, (b) is self-explanatory.

16 1 mole of an ideal gas is contained in a cubical volume, ABCDEFGH at 300 K (figure).
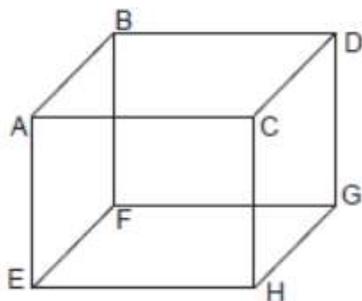
One face of the cube (EFGH) is made up of a material which totally absorbs any gas molecule incident on it. At any given time,

(a) the pressure on EFGH would be zero.

(b) the pressure on all the faces will the equal.

(c) the pressure of EFGH would be double the pressure on ABCD.

(d) the pressure on EFGH would be half that on ABCD.

Ans :

(d)

Since pressure is due to the total momentum of the gas molecules transferred to a face, its value is half for the face EFGH than that for face ABCD.

17 A cylinder containing an ideal gas is in vertical position and has a piston of mass M that is able to move up or down without friction figure. If the temperature is increased,

(a) both p and V of the gas will change.

(b) only p will increase according to Charle's law.

(c) V will change but not p.

(d) p will change but not V.

Ans :

(c) $\text{Pressure} = \dfrac{\text{Force}}{\text{Area}} = \dfrac{Mg}{\text{Area of the piston}} = \text{Constant}$

If the temperature is increased, only the volume increase as the piston moves up without friction. Recall $V \propto T$ at constant pressure.

18 Volume versus temperature graphs for a given mass of an ideal gas are shown in figure at two different values of constant pressure.   **1**

What can be inferred about relation between P_1 and P_2?

(a) $P_1 > P_2$

(b) $P_1 = P_2$

(c) $P_1 < P_2$

(d) data is insufficient

Ans : *(a)*

As $PV = RT$, $P = \dfrac{RT}{V}$ or $P \propto \dfrac{T}{V}$. thus, lesser the value of V for a given temperature, more is the value of P. Obviously, this is so in case pressure is P_1, i.e., $P_1 > P_2$.

19   For hydrogen gas, $C_P - C_V = b$. The relation between a and b is   **1**

(a) $a = 16b$

(b) $b = 16a$

(c) $a = b$

(d) $a = 4b$

Ans : (c)

For any gas, $C_P - C_V = R$

$$\therefore a = b$$

20   Avogadro's number is the number of molecules in   1

    (a) one litre of a gas at NTP.

    (b) one mole of a gas

    (c) one gram of a gas

    (d) one kg of a gas

Ans :(b)

21   Boyle's law is applicable for an   1

    (a) adiabatic process.

    (b) isothermal process.

    (c) isobaric process.

    (d) isochoric process.

Ans :

 (b)

Boyle's law is applicable to an isothermal process where temperature remains constant.

22   1 mole of H_2 gas is contained in a box of volume $V = 1.00 \ m_3$ at $T = 300$ K. The   1
    gas is heated to a temperature of $T = 3000$ K and the gas gets converted to a gas
    of hydrogen atoms. The final pressure would be (considering all gases to be
    ideal)

    (a) same as the pressure initially.

    (b) 2 times the pressure initially.

    (c) 10 times the pressure initially.

    (d) 20 times the pressure initially.

Ans :

(d)

At constant volume, $P \propto T$. When temperature becomes 10 times (from 300 K to 3000 K), P becomes 10 times. Further, since the pressure is due to the bombardment of particles, it doubles (i.e., becomes 2 times) due to two atoms (of a hydrogen molecule) than the molecule itself. The average KE of a gas particle (molecule/atom) depends only on its temperature and is given by $\frac{3}{2}k_B T$.

Thus, the pressure becomes 2×10 times, i.e., 20 times the initial pressure.

23 An inflated rubber balloon contains one mole of an ideal gas, has a pressure p,   **1**
volume V and temperature T. If the temperature rises to 1.1 T, and the volume is
increased to 1.05 V, the final pressure will be

(a) 1.1 p

(b) p

(c) less than p

(d) between p and 1.1.

Ans : *(d)*

As $pV = RT$ and $p'(1.05\ V) = R(1.1\ T)$.

$$p' = \left(\frac{1.1}{1.05}\right)\frac{RT}{V} = 1.0476p \left(As\ \frac{RT}{V} = p\right)$$

Thus p' lies between p and $1.1\ p$.

24 When pressure increases by 1%, what is the percentage decrease in the volume   **2**
of a gas, if Boyle's law is obeyed ?

Ans : We know PV = constant.
If P is increased by 1%, then volume will
decrease.

$\therefore$ (P + 0.01P) (V − dV) = PV

−PdV + 0.01PV − 0.01 PdV = 0

−PdV(1 + 0.01) = − 0.01 PV

$$\therefore \qquad \frac{dV}{V} = \frac{0.01}{1.01}$$

% variation in volume = 0.98

25   Explain the phenomenon of evaporation on the basis of kinetic theory.   **2**

Ans :

According to kinetic theory, molecules of a liquid are in a state of continuous
random motion. The molecules near the surface of liquid may have enough K.E.
so as to overcome the intermolecular attraction of other molecules on the surface
and hence manage to escape. Such molecules would move around freely in the
space above the liquid. This is the phenomenon of evaporation which may occur
at all temperatures.

26 Find the temperature at which root mean square velocity of a gas is half of its   **2**
value at 0°C, while pressure remaining constant.

Ans :

Since velocity is directly proportional to the square root of absolute temperature, we have,

$$v_{rms} \propto \sqrt{273+0} \text{ and } \frac{v_{rms}}{2} \propto \sqrt{273+t}$$

By dividing, we get,

$$2 = \sqrt{\frac{273}{273+t}}$$

Squaring both sides, we get,

$$4 = \frac{273}{273+t}$$

$$\Rightarrow \quad t = \frac{273}{4} - 273 = -204.75 \text{ °C}$$

27 Calculate the r.m.s velocity of oxygen molecule at 27°C. Given that atomic weight of oxygen is 16.   2

Ans : $c = \sqrt{3RT/M}$

$$= \sqrt{\frac{3 \times 1.98 \times 300}{16}} = 10.55 \text{ m/s}$$

28 Calculate the molecular K.E. of 1 gram of Helium (Molecular weight 4) at 127°C. Given R = 8.31 J mol^{-1} K^{-1}.   2

Ans : T = 127 + 273 = 400 K

Average K.E. per mole of Helium $= \frac{3}{2}$ RT

Average K.E. of 1 gram of Helium

$$= \frac{3}{2} \frac{RT}{M} = \frac{3 \times 8.31 \times 400}{2 \times 4}$$

$$= 1246.5 \text{ J.}$$

29 State the law of equipartition of energy and using this find the relation for the total internal energy of a mole of monoatomic gases.   2

Ans :

Law of equipartition of energy. For any dynamical system in thermal equilibrium, the total energy is distributed equally amongst all degrees of freedom and the energy associated with each molecule per degree of freedom is $\frac{1}{2}K_B T$, where K_B is Boltzmann constant and T is temperature of the system.

For monoatomic gas there are only three degrees of freedom.

For a gas in thermal equilibrium at temperature T, the average value of translational energy of molecule is

$$\langle E_t \rangle = \left\langle \frac{1}{2}mv_x^2 \right\rangle + \left\langle \frac{1}{2}mv_y^2 \right\rangle + \left\langle \frac{1}{2}mv_z^2 \right\rangle$$

Therefore energy associated with monoatomic molecule is $\frac{3}{2}K_B T$.

30 State the number of degrees of freedom possessed by a monoatomic molecule in space. Also give the expression for total energy possessed by it at a given   2

temperature. Hence give the total energy of the atom at 300K.

Ans :

A monoatomic gas has three degrees of freedom.

Energy associated with each molecule per degree of freedom = $\frac{1}{2}KT$.

Total energy of one monoatomic molecule at 300 K.

$= \frac{3}{2} \times 1.38 \times 10^{23} \times 300$

$= 3 \times 207 \times 10^{23}$ J $= 621 \times 10^{23}$ J

31 Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (mono-atomic), the second contains chlorine (diatomic) and third contains polyatomic gas. Do the vessels contains equal number of molecules ? Is the root mean square speed of molecules same in the three cases ?   **2**

Ans :

Yes, according to the Avogadro's hypothesis, number in each case is same.

No, *rms* speed of molecules is not same in three cases.

$\therefore \quad \frac{2v}{v} = \sqrt{\frac{T'}{27+273}} \quad \therefore v_{rms}$ of neon with minimum mass is maximum.

32 Calculate the temperature at which r.m.s. velocity of gas molecules become double than its value at 27°C, pressure of the gas remaining the same.   **2**

Ans : As P is constant and $v_{rms} \propto \sqrt{T}$, we have

$\therefore \qquad \frac{2v}{v} = \sqrt{\frac{T'}{27+273}}$

$\Rightarrow \qquad T' = 1200 \text{ K or } 927 \text{ °C}$

33 Prove that the average kinetic energy of a molecule of an ideal gas is directly proportional to the absolute temperature of the gas.   **3**

Ans : Pressure $P = \frac{1}{3} \frac{nmc^2}{V}$

Multiplying and dividing the right side by 2,

$PV = \frac{2}{3} \frac{1}{2} nmc^2 = \frac{2}{3} E$

where E is the average kinetic energy.

Also, PV = RT for 1 mole of a gas

$\therefore \qquad RT = \frac{2}{3} E$

$\therefore$ Kinetic energy (average) is directly proportional to the absolute temperature.

34 Figure shows plot of PV/T versus P for 1.0×10^{-3} kg of oxygen gas at two different temperatures.   **3**

(a) What does the dotted plot signify ?

(b) Which is true : $T_1 > T_2$ or $T_1 < T_2$?

(c) What is the value of PV/T where the curves meet on the y-axis ?

(d) If we obtained similar plots for 1.0×10^{-3} kg of hydrogen, would we get the same value of PV/T at the point where the curves meet on the y-axis ? If not, what mass of hydrogen yields the same value of PV/T (for low pressure high temperature region of the plot) ? (Molecular mass of $H_2 = 2.02$ u, $O_2 = 32.0$ u, $R = 8.31$ J mol^{-1} K^{-1}).

Ans :

(a) Since dotted plot is parallel to P-axis, it tells that value of $\dfrac{PV}{T}$ remains same, even when P is change i.e., it tells $\dfrac{PV}{T}$ constant.

(b) The curve at temperature T_1, is more close to the plot than the curve at temperature T_2. Since the behaviour of a real gas approaches the perfect gas behaviour as the temperature is increased. It follows that $T_1 > T_2$.

(c) If the amount of gas under consideration is 1 mole, then the value of $\dfrac{PV}{T}$, where the curves meet y-axis, will be R (= 8.31 J mol^{-1} K^{-1}). For oxygen, molecular mass is 32.0 g i.e., 32.0×10^{-3} kg. Since mass of oxygen gas under consideration is 1.0×10^{-3} kg, the value of $\dfrac{PV}{T}$, where the curve meet the y-axis is given by

$$\frac{PV}{T} = nR = \left(\frac{1}{32}\right) \times 8.31\,\text{JK}^{-1} = 0.26\,\text{JK}^{-1}$$

(d) If we obtain similar plots for 1.0×10^{-3} kg of hydrogen, the value of $\dfrac{PV}{T}$ where the curve meets the y-axis will not be same.
Since, molecular mass of hydrogen is 2.02×10^{-3} kg, mass of hydrogen that will yield same value of $\dfrac{PV}{T}$ i.e., 0.26 JK^{-1} will be $\dfrac{2.02 \times 10^{-3}}{8.31} \times 0.26 = 6.32 \times 10^{-5}$ kg.

35 Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic), and the third contains uranium hexafluoride (polyatomic). Do the vessels contain equal number of respective molecules ? Is the root mean square speed of molecules same in the three cases ? If not, in which case is vrms the largest ?   **3**

Ans :

Yes, because according to Avogadro's hypothesis. "Equal volume of all the gases contains equal number of molecules at the same temperature and pressure".

No, $V_{rms} = \sqrt{\dfrac{3KT}{M}}$ (M = mass of molecule)

Because M is different for different gases, therefore v_{rms} is also have different value.

36 Using the expression for pressure exerted by a gas, deduce Avogadro's law and Graham's law of diffusion.   **3**

Ans :

Avogadro's law : Consider two gases A and B having equal volume V at same temperature T and pressure P.

For gas A,

$$P = \frac{1}{3} \frac{M\rho_1}{V} c_1^2$$

For gas B,

$$P = \frac{1}{3} \frac{M_2}{V} c_2^2$$

$$\therefore \qquad P = \frac{M_1 c_1^2}{3V} = \frac{M_2 c_2^2}{3V}$$

or $\quad M_1 c_1^2 = M_2 c_2^2 \qquad\qquad ...(i)$

As gases are at the same temperature.

Average K.E. per molecule of gas A

$\qquad\qquad$ = Average K.E. per molecule of gas B

$$\frac{1}{2} m_1 c_1^2 = \frac{1}{2} m_2 c_2^2$$

$$m_1 c_1^2 = m_2 c_2^2 \qquad\qquad ...(ii)$$

Dividing (i) by (ii)

$$\frac{M_1 c_1^2}{m_1 c_1^2} = \frac{M_2 c_2^2}{m_2 c_2^2}$$

$$\frac{m_1 n_1 c_1^2}{m_1 c_1^2} = \frac{m_2 n_2 c_2^2}{m_2 c_2^2}$$

$$\therefore \qquad n_1 = n_2 \text{ (which is Avogadro's law)}$$

Graham's law :

$$P_1 = \frac{1}{3} \rho_1 c_1^2 \quad P_2 = \frac{1}{3} \rho_2 c_2^2$$

When the steady state of diffusion is reached,

$$P_1 = P_2$$

$$\frac{1}{3} \rho_1 c_1^2 = \frac{1}{3} \rho_2 c_2^2$$

$$\frac{c_1^2}{c_2^2} = \frac{\rho_2}{\rho_1} \text{ or } \frac{c_1}{c_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

If r_1 and r_2 are rates of diffusion of A and B respectively, then

$$r_1 \propto c_1, \, r_2 \propto C_2$$

$$\frac{r_1}{r_2} = \frac{c_1}{c_2}$$

or $\qquad\qquad \dfrac{r_1}{r_2} = \sqrt{\dfrac{\rho_2}{\rho_1}}$

$\Rightarrow \qquad\qquad r \propto \dfrac{1}{\sqrt{\rho}}$

37 Explain the concept of absolute zero of temperature on the basis of kinetic theory of gases. **3**

Ans :

Let us consider one gram mole of the gas. Let M and V be its mass and volume. The pressure exerted by the gas is given by

$$P = \frac{1}{3}m n \overline{v}^2 \qquad\qquad [M = mn]$$

$$PV = \frac{1}{3}n V m \overline{v}^2$$

$$PV = \frac{2}{3}N \times \frac{1}{2}m\overline{v}^2$$

But $\quad PV = RT$

$$RT = \frac{2}{3} N \times \frac{1}{2}m\overline{v}^2$$

$$\frac{1}{2}m\overline{v}^2 = \frac{3}{2}\frac{R}{N}T \qquad\qquad \left[\frac{R}{N} = K_B, \text{Boltzmann Constant}\right]$$

$$\frac{1}{2}m\overline{v}^2 = \frac{3}{2}K_B T$$

$$\overline{v} \propto \sqrt{T}$$

Hence, absolute temperature is the temperature at which the root mean square velocity of gas molecules reduces to zero.

38 Molar volume is the volume occupied by 1 mole of any (ideal) gas at standard temperature and pressure (S.T.P. : 1 atmospheric pressure, 0°C). Show that it is 22.4 litres. Take R = 8.31 J mol^{-1} K^{-1}. **4**

Ans : Here, $\quad$ P = 1 atm = 0.76 m of Hg

$\qquad\qquad = 0.76 \times (13.6 \times 10^3) \times 9.8$ Pa

$\qquad$ T = 273 K, R = 8.31 J mol^{-1} K^{-1}

$\qquad$ V = ? n = 1 mole

As, $\qquad PV = n$RT or V = $\dfrac{n\,RT}{P}$

$\therefore \qquad$ V = $\dfrac{1 \times 8.31 \times 273}{0.76 \times (13.6 \times 10^3) \times 9.8}$

$\qquad\qquad = 22.4 \times 10^{-3}$ m^3 = 22.4 litres

39 Calculate the temperature at which r.m.s. velocity of a gas molecule is the same as that of a molecule of another gas at 27°C. Molecular weight of first and second gases are 64 and 32 respectively. **4**

Ans : As per question; $c_1 = c_{27}$

or $\qquad c_0\sqrt{\dfrac{t+273}{273}} = c'_0\sqrt{\dfrac{27+273}{273}}$

or $\qquad \dfrac{c'_0}{c_0} = \sqrt{\dfrac{300}{t+273}}$

But $\qquad \dfrac{c'_0}{c_0} = \sqrt{\dfrac{M'_0}{M}} = \sqrt{\dfrac{32}{64}} = \sqrt{\dfrac{1}{2}}$

$\therefore \qquad \dfrac{300}{t+273} = \dfrac{1}{2}$ or t = 327 °C

40 Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 m³ at a temperature of 27°C and 1 atm pressure.   **4**

Ans :

$$V = 25.0 \text{ m}^3$$
$$T = 27 + 273 = 300 \text{ K}$$
$$K = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Now, $PV = nRT = n(NK)T$
$$= (nN)\, KT = N'KT$$

where $nN = N' = $ total no. of air bubbles in the given gas.

$$\therefore \quad N' = \frac{PV}{KT} = \frac{(1.01 \times 10^5) \times 25}{1.38 \times 10^{-23} \times 300}$$
$$= 6.10 \times 10^{26}$$

41 At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at – 20°C ? (atomic mass : Ar = 39.9 u, He = 4.0 u).   **4**

Ans :

Let c and c' be the rms velocity of Argon and Helium gas atom at temperature T K and T'K respectively.

$$M = 39.9, \ M' = 4.0, \ T = ?$$
$$T' = -20 + 273 = 253 \text{ K}$$

Now, $c = \sqrt{\dfrac{3RT}{M}} = \sqrt{\dfrac{3RT}{39.9}}$

and $c' = \sqrt{\dfrac{3RT'}{m'}} = \sqrt{\dfrac{3R \times 253}{4}}$

Since $c = c'$, therefore

$$\sqrt{\frac{3RT}{39.9}} = \sqrt{\frac{3R \times 253}{4}}$$

or $T = \dfrac{39.9 \times 253}{4} = 2523.7$ K

42 Estimate the mean free path and collision frequency of a nitrogen molecule in a cylinder containing nitrogen at 2.0 atm and temperature 17°C. Take the radius of the nitrogen molecule to be roughly 1.0 Å. Compare the collision time with the time the molecule moves freely between two successive collision (Molecular mass of N_2 = 28.0 u).   **4**

Ans :
$$PV = NK_BT$$
$$\therefore \quad n = \frac{N}{V}$$
$$= \frac{P}{K_BT} = \frac{2 \times 1.01 \times 10^5}{1.38 \times 10^{-23} \times 290}$$
$$\Rightarrow \quad n = 5 \times 10^{25} \text{ m}^{-3}$$

The mean, free path $\lambda = \dfrac{1}{\sqrt{2}\,\pi n\, d^2}$

where d is the diameter of the molecule.

$$\because \quad r = 1.0 \text{ Å}$$
$$\therefore \quad d = 2.0 \text{ Å} = 2 \times 10^{-10} \text{ m}$$
$$\therefore \quad \lambda = \frac{1}{1.414 \times \pi \times 5 \times 10^{25} \times (2 \times 10^{-10})^2}$$

or, $\quad \lambda = 1.12 \times 10^{-7}$ m

or, the $r.m.s.$ speed is given by

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 290}{28 \times 10^{-18}}}$$
$$= 5.08 \times 10^2 \text{ m/sec}$$

The frequency of collision
$$= \text{no. of collisions per second}$$
$$= \frac{v_{rms}}{\lambda} = \frac{5.08 \times 10^2}{1.12 \times 10^{-7}}$$
$$= 4.53 \times 10^9 \text{ Hz}$$

43 A balloon partially filled with Helium has a volume of 30 m³, at the earth's surface, where pressure is 76 cm of Hg and temperature is 27°C. What will be the increase in volume of gas if balloon rises to a height, where pressure is 7.6 cm of Hg and temperature is –54°C ?   **4**

Ans :
$$\frac{P_1 V_1}{T_1} = \frac{PV}{T}$$

or $\quad V_1 = \dfrac{PVT_1}{TP_1}$

$$= \frac{76 \times 30 \times (273 - 54)}{(273 + 27) \times 7.6}$$
$$= 219 \text{ m}^3$$

Hence increase in volume
$$= V_1 - V$$
$$= 219 - 30$$
$$= 189 \text{ m}^3$$

44 Derive an expression for pressure of a gas in a container. Using it, relate K.E. with pressure.   **5**

Ans :

1. The kinetic theory of gases is based on the following assumptions :

(i) A gas consists of a very large number of molecules which are perfect elastic spheres are identical in all respects for a given gas and are different for different gases.

(ii) The molecules of a gas are in a state of continuous, rapid and random motion in all directions with different speeds, ranging from zero to infinity and obey Newton's laws of motion.

(iii) The size of the gas molecules is very small as compared to the distance between them. Hence volume occupied by the molecules is negligible in comparison to the volume of the gas.

(iv) The molecules do not exert any force of attraction or repulsion on each other, except during collision.

(v) The collisions of the molecules with themselves and with the walls of the vessels are perfectly elastic. i.e., the momentum and the kinetic energy of the molecules are conserved during collisions.

Expression for pressure due to an ideal gas :

Consider an ideal gas contained in a cubical container OPQRSTKL of each side a and having a volume V.

Clearly, volume of the gas, V = volume of the container = a^3 i.e., V = a^3

Let there be n molecules of the gas in the container each of mass m. Then total mass of the gas in the container is

$$M = m \times n$$

Let the random velocities of the gas molecules A_1, A_2, ... A_n be c_1, c_2, ... c_n respectively. Let (x_1, y_1, z_1), (x_2, y_2, z_2), ... (x_n, y_n, z_n) be the rectangular components of the velocities c_1, c_2, ... c_n respectively, along three mutually perpendicular directions OX, OY and OZ.

$$x_1^2 + y_1^2 + z_1^2 = c_1^2$$
$$x_2^2 + y_2^2 + z_2^2 = c_2^2$$
$$\dots\dots\dots\dots\dots\dots\dots$$
$$x_n^2 + y_n^2 + z_n^2 = c_n^2$$

Change in momentum of A_1 along OX after one collision = $- mx_1 - mx_1 = - 2mx_1$

According to law of conservation of linear momentum, in one collision, the momentum transferred to the wall QRSL by the molecule A_1 will be = $+ 2 mx_1$

The molecule A_1 returned from face QRSL, collides against the wall OPKT, rebounds and again collides with QRSL. Thus the molecule A_1 covers a distance $2a$ between two successive collisions with the wall QRSL.

$\therefore$ Time between two successive collisions, $t = \dfrac{\text{distance}}{\text{speed}} = \dfrac{2a}{x_1}$

It means the molecule A_1 will collide with wall QRSL after every $\dfrac{2a}{x_1}$ seconds. Therefore, number of collisions per second with wall QRSL $= \dfrac{1}{t} = \dfrac{x_1}{2a}$

$\therefore$ Momentum transferred to the wall QRSL in one second by molecule A_1 = momentum transferred in one collision × no. of collisions in 1 sec.

$$= 2mx_1 \times \frac{x_1}{2a} = \frac{mx_1^2}{a}$$

According to Newton's 2nd law of motion, the rate of change of momentum of a body is equal to the force exerted on it. Hence the force exerted by the molecule A_1 on the wall QRSL will be,

$$f_1 = \frac{mx_1^2}{a}$$

$$f_2 = \frac{mx_2^2}{a}, \; ..... \; f_n = \frac{mx_n^2}{a}$$

Total force exerted by all the molecules on wall QRSL will be

$$F_x = \frac{mx_1^2}{a} + \frac{mx_2^2}{a} + ... + \frac{mx_n^2}{a}$$

$$= \frac{m}{a}(x_1{}^2 + x_2{}^2 + x_n{}^2)$$

$\therefore$ Pressure exerted on the wall QRSL is

$$P_x = \frac{\text{force}}{\text{area of wall QRSL}}$$

$$= \frac{F_x}{a^2}$$

$$= \frac{m}{a^3}(x_1{}^2 + x_2{}^2 + ... + x_n{}^2)$$

Similarly, the pressure exerted by the gas molecules perpendicular to OY and OZ respectively are given by

$$P_y = \frac{m}{a^3}(y_1{}^2 + y_2{}^2 + ... + y_n{}^2)$$

and $\qquad P_z = \dfrac{m}{a^3}(z_1{}^2 + z_2{}^2 + ... + z_n{}^2)$

Since the molecular density is uniform throughout the gas, therefore the pressure exerted by the molecules is the same in all directions. Hence,

$$P_x = P_y = P_x = P(\text{say})$$

$\therefore \qquad P_x + P_y + P_z = 3P$

or $\qquad\qquad P = \dfrac{P_x + P_y + P_z}{3}$

$$P = \frac{1}{3}\left[\frac{m}{a^3}(x_1^2 + x_2^2 + ... + x_n^2) + \frac{m}{a^3}(y_1^2 + y_2^2 + ... + y_n^2) + \frac{m}{a^3}(z_1^2 + z_2^2 + ... + z_n^2)\right]$$

$$= \frac{m}{3a^3}\left[(x_1^2 + x_2^2 + ... + x_n^2) + (y_1^2 + y_2^2 + ... + y_n^2) + (z_1^2 + z_2^2 + ... + z_n^2)\right]$$

$$P = \frac{m}{3a^3} [(x_1^2 + y_1^2 + z_1^2) + (x_2^2 + y_2^2 + z_2^2) + \ldots + (x_n^2 + y_n^2 + z_n^2)]$$

$$= \frac{m}{3V} [c_1^2 + c_2^2 + \ldots + c_n^2]$$

$$= \frac{1}{3} \frac{mn}{V} \left[\frac{c_1^2 + c_2^2 + \ldots + c_n^2}{n} \right]$$

$$P = \frac{M}{3V} c^2$$

where $c = \sqrt{\dfrac{c_1^2 + c_2^2 + \ldots + c_n^2}{n}}$ is called the root mean square $(r.m.s.)$ velocity of the gas molecules.

Relation of K. E. with pressure :

Pressure $P = \dfrac{1}{3} \dfrac{nmc^2}{V}$

$\Rightarrow \qquad P = \dfrac{1}{3} \rho c^2 \qquad \left[\because \dfrac{m}{v} = \rho \right]$

Mean K.E. of translation per unit volume of gas is $E = \dfrac{1}{2} \rho c^2$

$\therefore \qquad \dfrac{P}{E} = \dfrac{1/3\, \rho c^2}{1/2\, \rho c^2} = \dfrac{2}{3}$

$$\boxed{P = \frac{2}{3} E}$$

45 Define degrees of freedom. How will you account for the five degrees of freedom in a diatomic molecule ? Using the idea of equi-partition of energy, find the value of γ for the same.   **5**

Ans :

The number of degrees of freedom of a dynamical system is defined as total number of coordinates required to describe completely the position and configuration of the system.

The diatomic molecules have two atoms in it. The molecule is capable of translatory motion

of its centre of mass and also it can rotate about its centre of mass. Therefore it has three translational degrees of freedom and two rotational degrees of freedom.

So diatomic molecule has in all five degrees of freedom.

Using

law of equipartition of energy, total energy of one mole of diatomic molecule is

$$U = 5 \times \left(\frac{1}{2}K_B T\right) \times N_A = \frac{5}{2}RT$$

$$C_v = \left(\frac{dU}{dT}\right)$$

$$C_v = \frac{d}{dT}\left(\frac{5}{2}RT\right) = \frac{5}{2}R$$

$$C_v = 4.96 \text{ cal mol}^{-1} \text{ K}^{-1}$$

also,
$$C_P = C_v + R = \frac{5}{2}R + R = \frac{7}{2}R$$

$$C_P = \frac{7}{2} \times 1.985$$

$$C_P = 6.95 \text{ cal mol}^{-1} \text{ K}^{-1}$$

$$\gamma = \frac{C_P}{C_v} = \frac{7/2\ R}{5/2\ R} = 1.4$$